Advances in Asset Allocation

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Deconstructing Black-Litterman:
How to Get the Portfolio You Already Knew You Wanted

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Preface

- Black-Litterman (BL) superficially attractive
  - CAPM, market equilibrium, Bayesian, reverse returns
- CFA teaches BL and Michaud
  - Competing alternatives for solving Markowitz
- BL taught and used for 20 years and counting
  - Academics, professional journals, many 100s billions AUM
- NFA conclusions
  - BL is no better than Markowitz
  - BL possesses serious mathematical, statistical, financial errors
  - It can’t be recommend for practice
  - Implications for CFA curriculum
  - Represents a financial enigma

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Overview

- Markowitz optimization promise and limitations
- Black-Litterman (BL) proposal to solve Markowitz
  - Mathematical BL framework analysis
  - Illustrate BL
  - Compare BL to Markowitz and Michaud optimization
- Additional BL limitations
- Conclude

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Markowitz Mean-Variance (MV) Optimization

- Seminal paper published March 1952
  - Sixty years later often not well understood
  - No normal distribution or specialized utility required

- Markowitz objective
  - Mathematical framework describing investor behavior

- Need to model
  - Pricing reflects expectation of return with vague notion of risk.
  - Well diversified institutional portfolios that vary by risk level

- Markowitz insight:
  - Sign constrained MV optimization consistent investor behavior
  - No need of fat tails, tail risk, copulas, black swans, etc.
  - A convenient framework for computing optimal portfolios consistent with how securities are priced in many capital markets

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Markowitz Mean-Variance (MV) Optimization

- A theoretical standard for defining portfolio optimality
  - Central to modern finance and investment theory

- But practice is very different
  - Very unstable
    - small changes in inputs lead to large optimized changes
  - Poor diversification, hard to manage, poor performance

- Ten asset classes input to MV efficient frontier
  - Money market, intermediate fixed, long-term fixed, High yield, small cap value, small cap growth, large cap value, large cap growth, international equity, real estate
  - Thirty years of historical monthly returns

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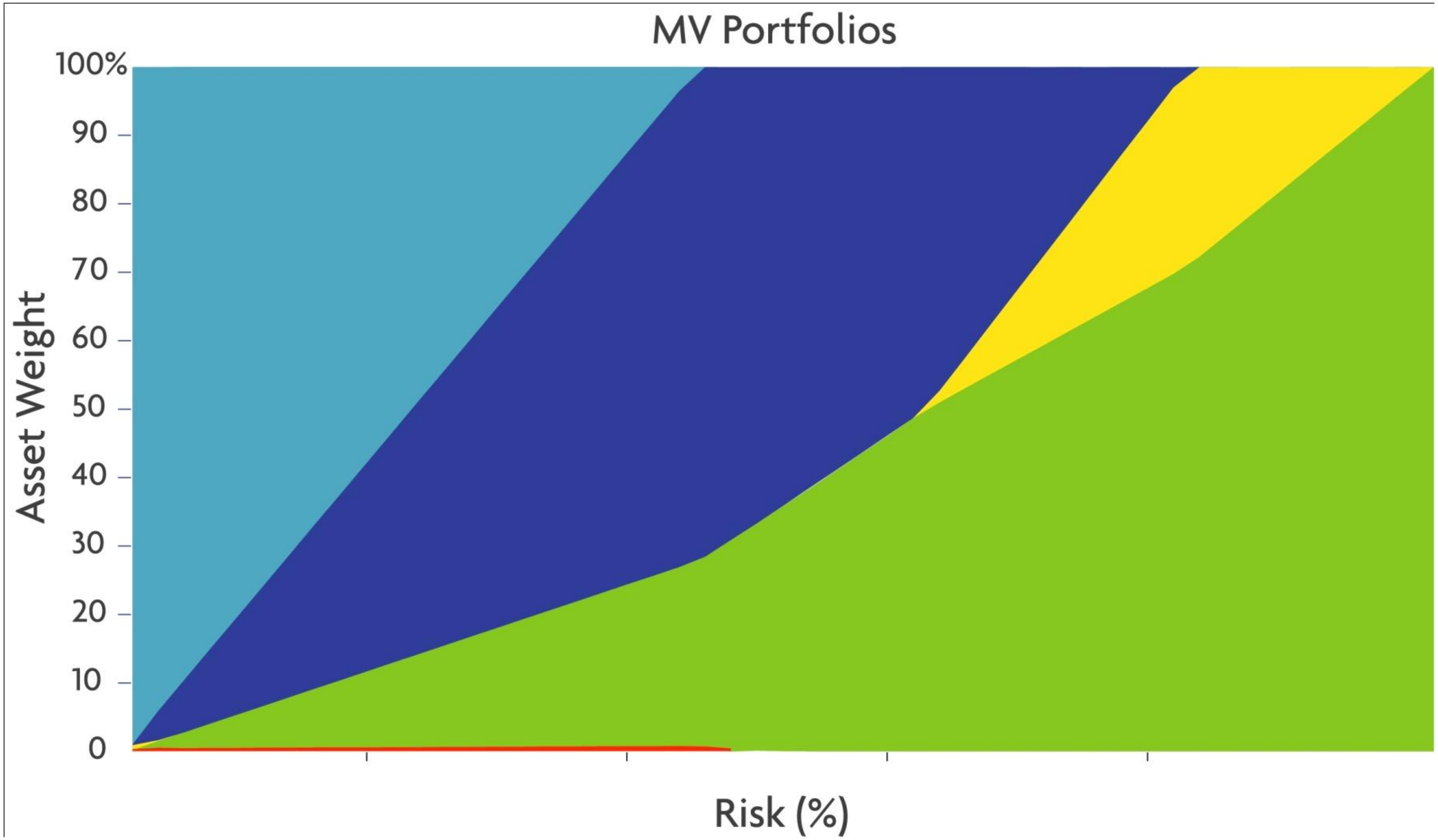
MV Efficient Frontier Composition Map:
10 Asset Example



Asset Allocation In Practice

- Manage the inputs
- Constrain the solution
- Engineer “acceptable” portfolios

- Why bother optimizations (Michaud 1989)
  - Managers/consultants ignore MV optimization
  - Disguised active management

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Black-Litterman Optimization: Proposal to Solve MV Limitations

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Black-Litterman (BL) Optimization

- Assume a “market” portfolio in “equilibrium” on inequality unconstrained MV frontier
  - Implies max Sharpe ratio (MSR) optimal
- Estimate covariance matrix and “reverse” optimize to estimate returns
  - “Implied” or “reverse” returns make market optimal
- Overlay reverse returns with investor views (Bayes)
- Compute the BL optimal portfolio

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BL Potential Benefits

- Result is generally easy to understand and stable
  - BL optimal typically close to market portfolio

- Reflects how many traditional managers and investment committees think
  - Start with a “good” portfolio and tweak it

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Black-Litterman Mathematical Framework

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Assumptions and Notation

- **M: Assume "Market" portfolio in equilibrium**
  - **MSR on inequality unconstrained MV frontier**
- **$\sum$: Covariance matrix typically estimated from data**
  - **No estimation error assumed**
- **Usual unconstrained MSR optimization formula: $X=\sum^{-1}\mu$**
  - **μ is estimated return, X is the MSR optimal portfolio**
- **Inverse or implied returns formula: $\Pi=\sum M$:**
- **Equilibrium returns implicitly assume $N(\Pi,\tau\sum)$ distribution**
  - **But Π is a point estimate of what?**
  - **$\tau\sum$ does not characterize information in Π**
  - **Setting for scale parameter τ unclear**

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Assumptions and Notation

- $\mathbf{P}$: collection of view portfolios
- $\mathbf{P}\boldsymbol{\mu} \sim N(\boldsymbol{\nu}, \boldsymbol{\Omega})$: Investor views
- Information from separate normal distributions (views and equilibrium returns) are combined into one with Theil-Goldberger (1961) formula

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Black-Litterman Mean with Views

- Combined means $\boldsymbol{\mu}_{BL}$

$$= \boldsymbol{\Pi} + \mathbf{V}$$

$$= \boldsymbol{\Pi} + \boldsymbol{\Sigma}\mathbf{P}' \left(\frac{\boldsymbol{\Omega}}{\tau} + \mathbf{P}\boldsymbol{\Sigma}\mathbf{P}'\right)^{-1} (\boldsymbol{v} - \boldsymbol{P}\boldsymbol{\Pi})$$

- Note: the *ad hoc* τ parameter scales relative importance of views in the Theil-Goldberger formula

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Black-Litterman Portfolio

- $$\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}_{\mathrm{BL}} = \underbrace{\mathbf{M}}_{N \times 1} + \underbrace{\mathbf{P}'\left(\frac{\boldsymbol{\Omega}}{\tau} + \mathbf{P\Sigma P}'\right)^{-1}(\boldsymbol{\nu} - \boldsymbol{P\Pi})}_{N \times 1}$$

- Deviates from market weights only assets with views
- Temptation to change results by changing τ

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Discussion

- Solely a function of market portfolio and views
- Σ is only source of data in formula
    - Essentially no risk-return information from data!
- Implied returns designed solely for MSR point
    - Extension to full efficient frontier gives unpredictable/unstable results
- Extension to constraints illustrated in next sections
- Inconsistent with principles of good data analysis or scientific method

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Illustrating Black-Litterman

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Start with Risk-Return Estimates

Example: Michaud (1998, Ch. 2)

Asset Name	Mean	Std Dev		Euro Bonds	US Bonds	Canada	France	Germany	Japan	UK	US
Euro Bonds	3.22%	5.40%		1.00	0.92	0.33	0.26	0.28	0.16	0.29	0.42
US Bonds	2.96%	6.98%		0.92	1.00	0.26	0.22	0.27	0.14	0.25	0.36
Canada	4.64%	19.04%		0.33	0.26	1.00	0.41	0.30	0.25	0.58	0.71
France	10.53%	24.36%		0.26	0.22	0.41	1.00	0.62	0.42	0.54	0.44
Germany	6.36%	21.55%		0.28	0.27	0.30	0.62	1.00	0.35	0.48	0.34
Japan	10.53%	24.37%		0.16	0.14	0.25	0.42	0.35	1.00	0.40	0.22
UK	9.53%	20.83%		0.29	0.25	0.58	0.54	0.48	0.40	1.00	0.56
US	8.53%	14.89%		0.42	0.36	0.71	0.44	0.34	0.22	0.56	1.00

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Define a "Market" Portfolio

Example: 60/40 Equal Weighting of Bonds and US and non-US equities

Asset Name	Market Portfolio	Mean	Standard Deviation
Euro Bonds	20.0%	3.2%	5.4%
US Bonds	20.0%	3.0%	7.0%
Canada	6.0%	4.6%	19.0%
France	6.0%	10.5%	24.4%
Germany	6.0%	6.4%	21.5%
Japan	6.0%	10.5%	24.4%
UK	6.0%	9.5%	20.8%
US	30.0%	8.5%	14.9%

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BL Means

Compute "Implied" Returns that make Market MV Optimal

Asset Name	Market Portfolio	Original Means	Standard Deviation	BL Mean
Euro Bonds	20.0%	3.2%	5.4%	2.2%
US Bonds	20.0%	3.0%	7.0%	2.6%
Canada	6.0%	4.6%	19.0%	9.2%
France	6.0%	10.5%	24.4%	10.9%
Germany	6.0%	6.4%	21.5%	8.6%
Japan	6.0%	10.5%	24.4%	7.8%
UK	6.0%	9.5%	20.8%	10.0%
US	30.0%	8.5%	14.9%	8.5%

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Comment On BL Means

- Estimation error issues are ignored
  - No way to check how close market is to MSR optimality
  - Estimation errors in covariance ignored

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Add Investor Views

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Add Investor Views

Example: US vs. European Equities

Asset Name	Market	Mean	Std Dev	BL Mean	Investor Views
Euro Bonds	20.0%	3.2%	5.4%	2.2%	0.0%
US Bonds	20.0%	3.0%	7.0%	2.6%	0.0%
Canada	6.0%	4.6%	19.0%	9.2%	0.0%
France	6.0%	10.5%	24.4%	10.9%	-40.0%
Germany	6.0%	6.4%	21.5%	8.6%	-30.0%
Japan	6.0%	10.5%	24.4%	7.8%	0.0%
UK	6.0%	9.5%	20.8%	10.0%	-30.0%
US	30.0%	8.5%	14.9%	8.5%	100.0%
			View Prior Return		5.0%
			View Prior Std Dev		5.0%

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BL View Means Formula

- Uses Theil-Goldberger (1961) "contrast" procedure
  - Bayes method for adding exogenous information
  - Applicable with any risk-return estimates
  - Powerful asset management tool
  - Part of our software and investment process

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BL View Means

View Modifies BL Means

Asset Name	Market Portfolio	Mean	Standard Deviation	BL Mean	BL View Mean	Investor Views
Euro Bonds	20.0%	3.2%	5.4%	2.2%	2.2%	0.0%
US Bonds	20.0%	3.0%	7.0%	2.6%	2.6%	0.0%
Canada	6.0%	4.6%	19.0%	9.2%	9.6%	0.0%
France	6.0%	10.5%	24.4%	10.9%	5.5%	-40.0%
Germany	6.0%	6.4%	21.5%	8.6%	3.8%	-30.0%
Japan	6.0%	10.5%	24.4%	7.8%	4.9%	0.0%
UK	6.0%	9.5%	20.8%	10.0%	7.3%	-30.0%
US	30.0%	8.5%	14.9%	8.5%	10.0%	100.0%

View Return	5.0%
View Std Dev	5.0%
Data Return	-1.5%
Data Std Dev	16.7%

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BL Optimal Portfolio Relative to Views

Asset Name	Market	BL View Means	BL Optimal
Euro Bonds	20.0%	2.2%	20.0%
US Bonds	20.0%	2.6%	20.0%
Canada	6.0%	9.6%	6.0%
France	6.0%	5.5%	-6.5%
Germany	6.0%	3.8%	-3.4%
Japan	6.0%	4.9%	6.0%
UK	6.0%	7.3%	-3.4%
US	30.0%	10.0%	61.2%
Return	6.1%		7.2%
Risk	9.6%		10.3%

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Black-Litterman τ-Adjustment

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The τ-adjustment for BL Optimal

- BL introduce τ-adjustment of investor views
  - Allows computation of sign constrained BL optimal portfolios

- Define BL* the least sign constrained BL optimal portfolio
- Need to compute the value of τ: $0 \le \tau \le 1$
- Impact is to reduce certainty in investor views
- Reducing certainty in views implies BL optimal closer to M

- In this case the τ value is 0.07027888706
- Implies the view has a 18.8607% uncertainty std dev
- BL* portfolio can be computed directly with view modified means

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BL* Sign Constrained Optimal: τ-Adjusted Inputs

Asset Name	Market	BL View Means	BL Optimal	BL* Means	BL* Optimal
Euro Bonds	20.0%	2.2%	20.0%	2.2%	20.0%
US Bonds	20.0%	2.6%	20.0%	2.6%	20.0%
Canada	6.0%	9.6%	6.0%	9.4%	6.0%
France	6.0%	5.5%	-6.5%	8.3%	0.0%
Germany	6.0%	3.8%	-3.4%	6.3%	1.5%
Japan	6.0%	4.9%	6.0%	6.4%	6.0%
UK	6.0%	7.3%	-3.4%	8.7%	1.5%
US	30.0%	10.0%	61.2%	9.2%	45.0%
Return	6.1%		7.2%		6.2%
Risk	9.6%		10.3%		9.6%

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Black-Litterman and Markowitz

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BL* is Markowitz

- BL* is a sign constrained MSR optimal portfolio on the unconstrained MV efficient frontier

- Compute a Markowitz MSR sign constrained optimal portfolio
  - BL* means and Σ
  - Same inputs, same criteria as BL*
  - Same portfolio!

- BL* just a point on the Markowitz efficient frontier
  - BL* is nothing new!
  - BL* inherits all of Markowitz optimization limitations!
  - Input estimation error and instability unresolved

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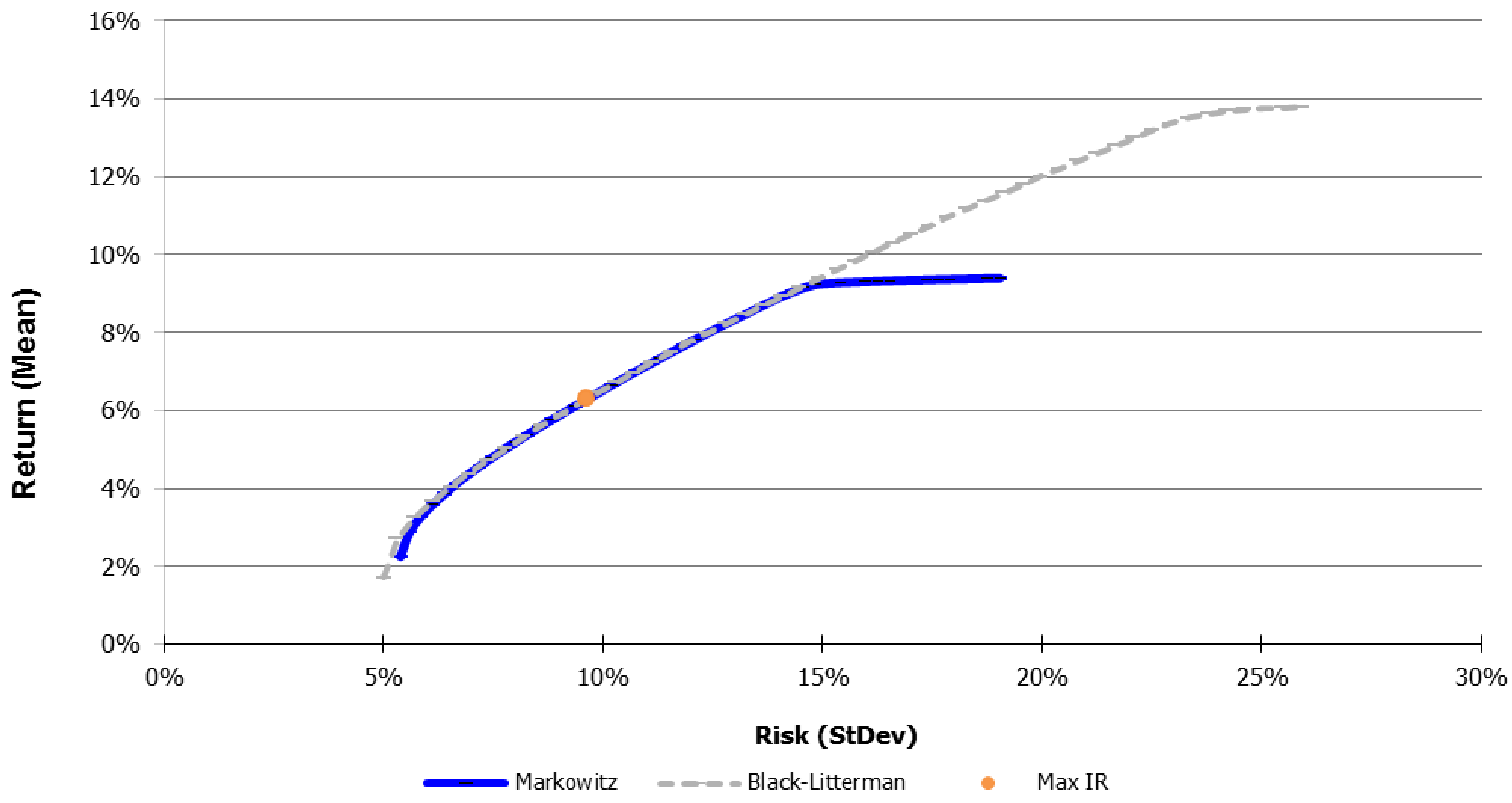
Black-Litterman and Markowitz Frontiers

BL* Inputs



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Michaud Optimization:
Patented Alternative to Markowitz

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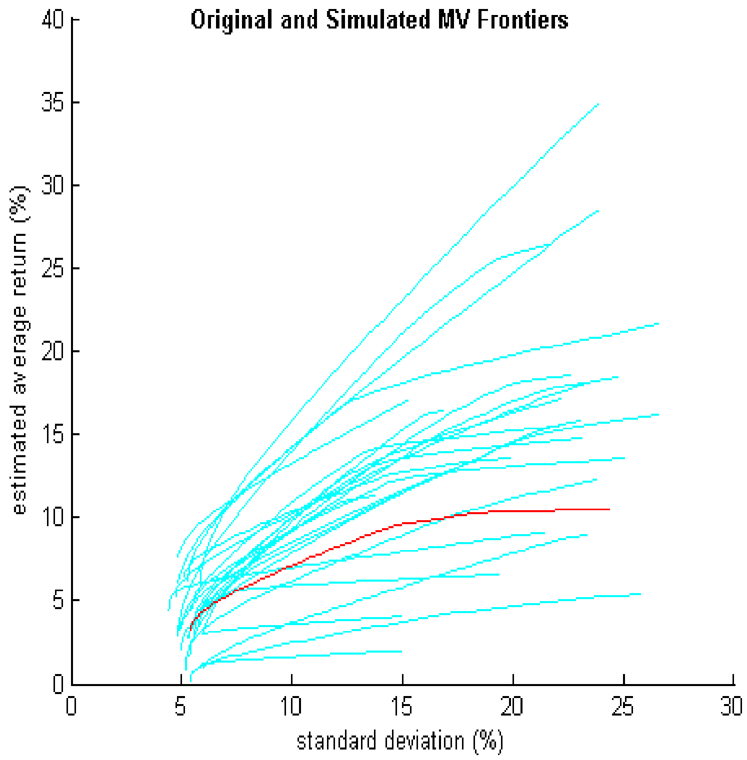
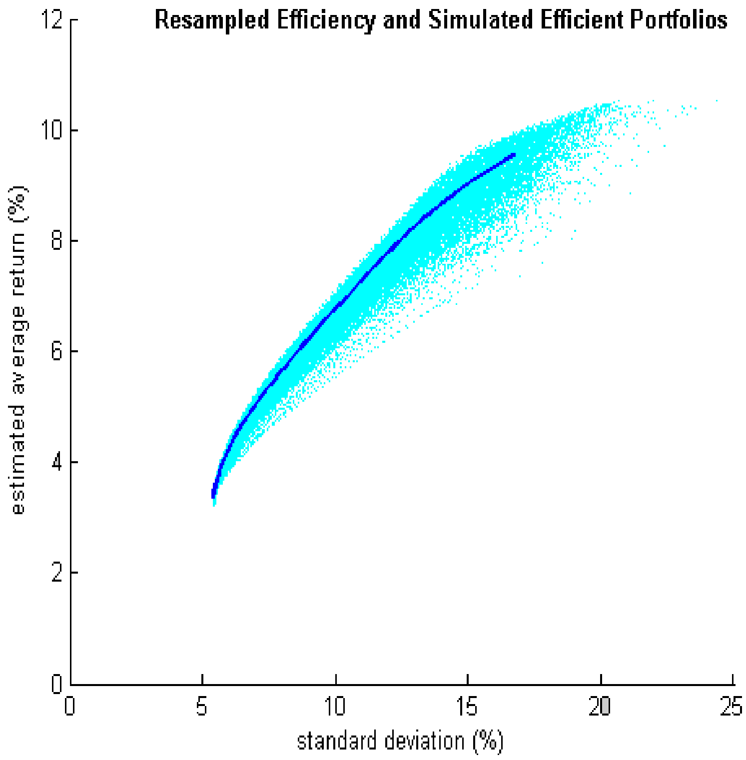
MV Resampling and Michaud Frontier





Compare BL* to Michaud MSR

BL* vs. Markowitz vs. Michaud

- Compute Michaud efficient frontier with same inputs as BL*
- Compare BL*/Markowitz MSR vs. Michaud MSR
- For sign constrained optimization
  - No reason to focus only on MSR optimal
  - Compare BL*/Markowitz vs. Michaud frontier allocations
  - Michaud better diversified across efficient frontier

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MSR BL*/Markowitz vs. Michaud

Asset Name	Market	BL*/Markowitz	Michaud
Euro Bonds	20.0%	20.0%	23.0%
US Bonds	20.0%	20.0%	19.9%
Canada	6.0%	6.0%	9.9%
France	6.0%	0.0%	4.3%
Germany	6.0%	1.5%	4.7%
Japan	6.0%	6.0%	6.6%
UK	6.0%	1.5%	5.4%
US	30.0%	45.0%	26.2%
Return	6.1%	6.2%	5.9%
Risk	9.6%	9.6%	9.3%

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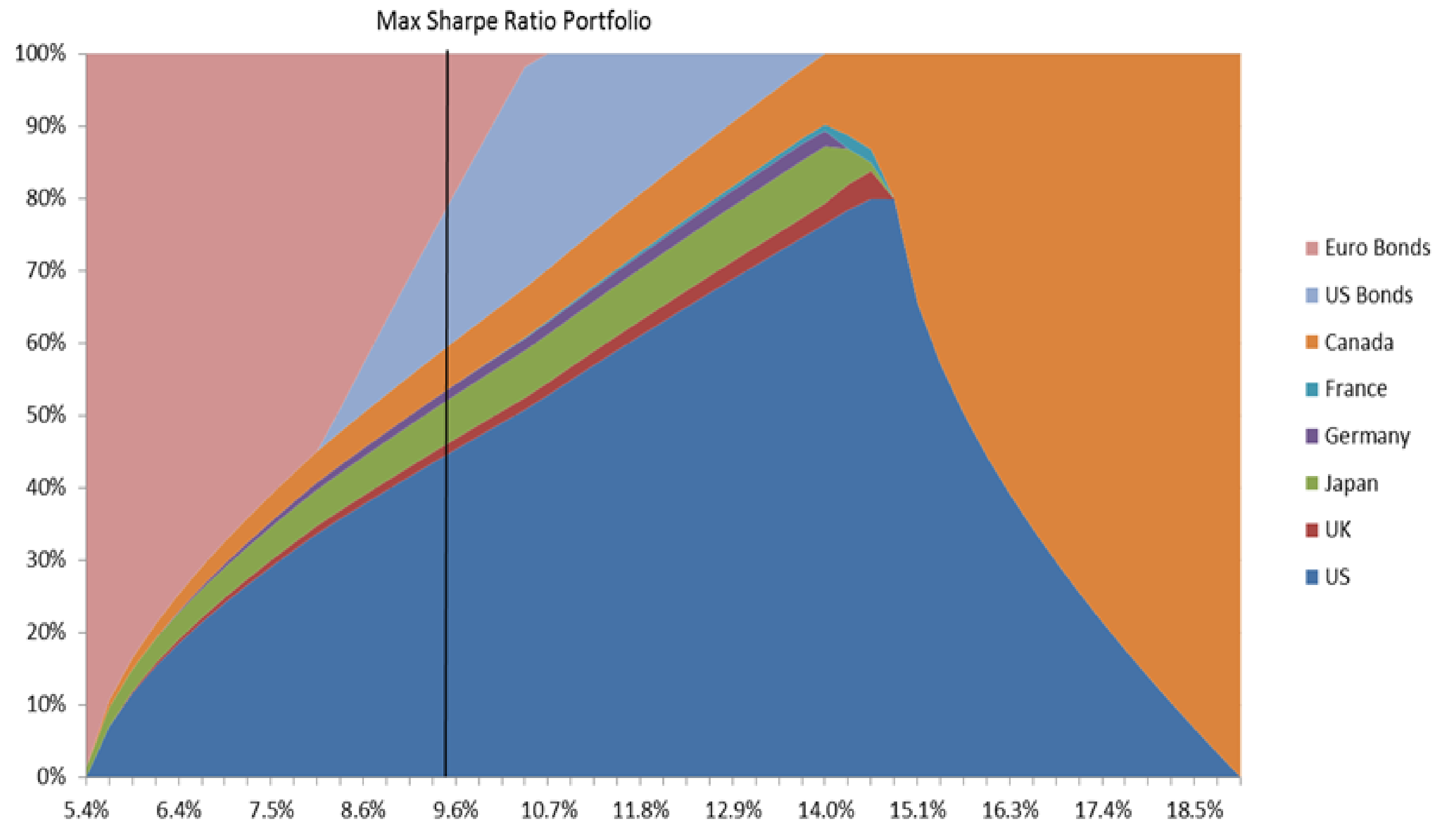
BL * Inputs Sign Constrained Markowitz Frontier Composition Map



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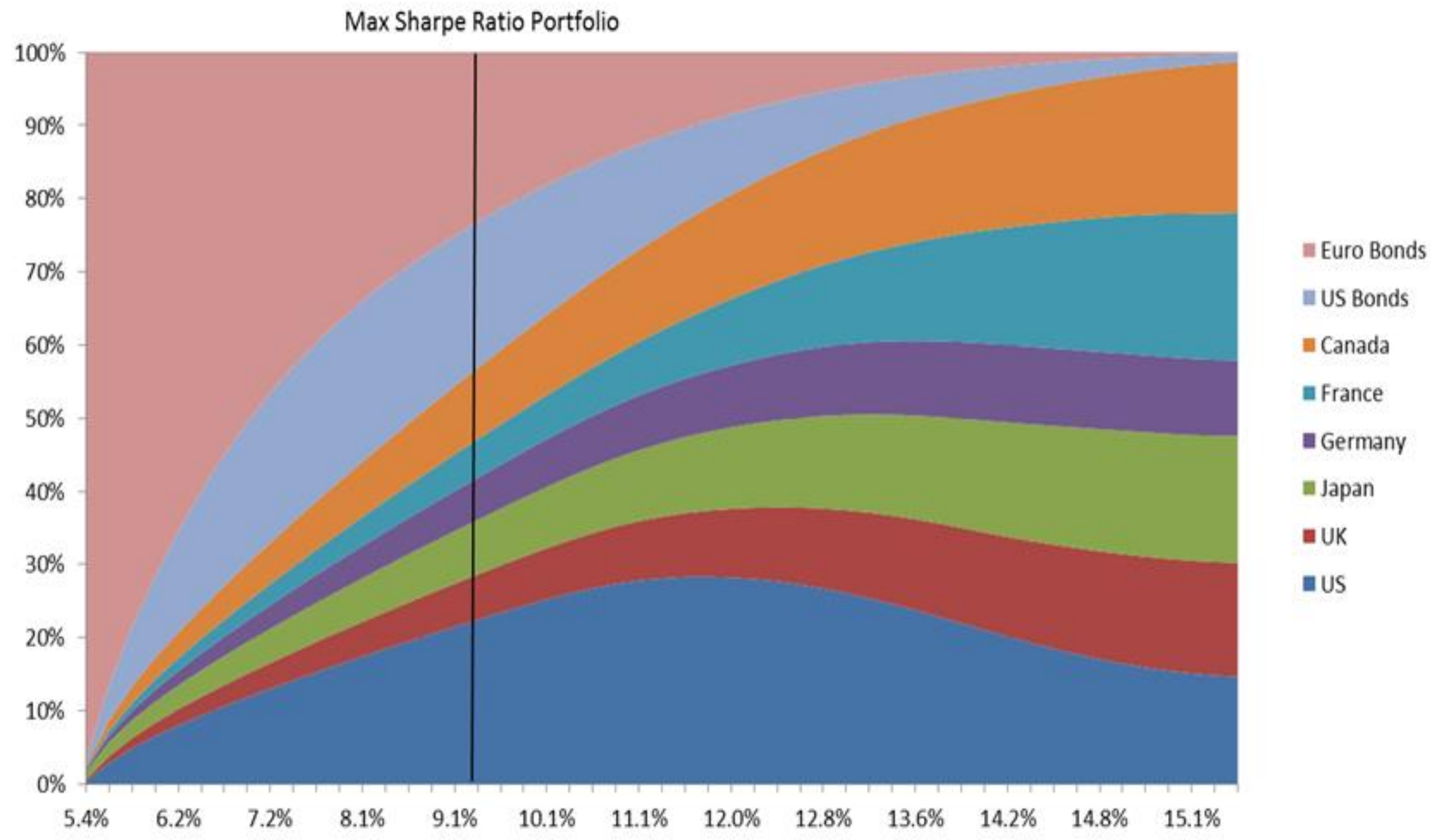
BL* Inputs Sign Constrained Michaud Frontier Composition Map



Additional BL Issues

Market Assumption Drives BL

- Equilibrium market is unknown and indefinable
- BL “market” portfolio has much unknown estimation error
- Implied returns makes any crazy portfolio MV “optimal”
- No constraint on investment reality
- Very dangerous investing
- Essentially ignoring contact with capital market reality
- Does not resolve estimation error in MV optimization

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BL is Unconstrained MV Optimization

- Jobson and Korkie (1980, 1981) prove that MSR portfolio on the unconstrained MV efficient frontier has little, if any, investment value
  - Much worse than equal weighting

- Regulatory restrictions and institutional limitations reflect constraints that are real world considerations in defining optimality
  - Proper linear constraints necessary condition for defining effective portfolio optimality

- Markowitz (2005) demonstrates linear constraints alter the validity of tools and theorems of modern finance

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BL ignores Investor Risk Aversion

- BL computes a single optimal portfolio
- Wide consensus that choice of portfolio risk the single most important investment decision
- BL portfolio risk often investor inappropriate
- Efficient frontiers required to provide a range of optimal risk habitats

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Statistical Foundations of BL Returns

- No standard statistical procedure based on inference from data produces the implied returns as estimates for portfolio expectations

- τ-adjustment compromises the character of an exogenous prior
  - Exogenous prior should not be changed
  - BL a major departure from rigorous Bayesian analysis

- Implied returns not investment useful
  - Returns not unique or on same scale as forecast returns
  - Implied means are estimated by covariances which have no information on the means by definition
  - Covariance has a great deal of estimation error
  - Covariance simulations show implied returns all over the map
  - Usual meanings of mean and variance have been lost

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Summary

Black-Litterman propose to solve MV optimization instability

However…

- Ad hoc “market” assumption drives process
- Much estimation error of true market portfolio likely
- Estimation error in covariance matrix ignored
- BL* no different than Markowitz for same inputs
- Inherits instability of Markowitz optimization
- Ignores investor risk habitat preferences
- Michaud optimization superior diversification for same inputs

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Conclusion

- BL optimization
  - No different from Markowitz in sign constrained case
  - Heir to Jobson-Korkie problems in unconstrained case
  - Inconsistent with modern statistical inference
  - Disconnected from capital market reality
  - Promotes very dangerous investing practices

- Effective asset management requires
  - Inequality constrained optimization
  - Frontier of efficient risk managed portfolios
  - Input estimation consistent with modern inference
  - Estimation error effective optimization

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Thank You

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