Navigating Canada's Factor Zoo

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ABSTRACT

This study delves into the battle of factors in Canadian capital markets, employing spanning tests to evaluate 17 factors from ten multifactor models for 1991–2022. While the value factor (HML) proves redundant, its monthly updated counterpart excels. The size factor (SMB) is not improved by discounting mispriced stocks but gains potency after controlling for profitability and investment. Q-based and mispricing factors subsume the momentum factor (UMD). No single asset-pricing model emerges dominant, except in three instances. A six-factor model including market, size, monthly updated value, ROE, expected growth, and PEAD factors proves effective for asset pricing in Canadian markets.

Keywords: factors spanning tests, Canadian stock markets, multifactor asset-pricing models

JEL Classifications: G11, G12, G14, G15

1. Introduction

Asset pricing factors are essential tools for portfolio managers and investment management practitioners, offering valuable insights into the underlying drivers of asset returns, security selection, and portfolio construction. Interestingly, scholarly exploration of asset-pricing factors commanding return premiums in capital markets traces its origin back to the early 1960s. Initially formulated by Sharpe (1964) and Lintner (1965), the capital asset-pricing model (CAPM) posits that only sensitivity to the market factor determines differences in expected returns among financial securities. However, empirical research conducted in the 1980s and the 1990s brought forth numerous anomalies that challenged this prediction.[1] A paradigm shift occurred when Fama and French (1993) introduced factors related to firm size (SMB) and value (HML), resulting in a three-factor asset-pricing model (TFPM). This development continued with the emergence of the five-factor model (FF5) of Fama and French (2015), which incorporates profitability (CMA) and investment (RMW) as additional factors. Since then, the landscape of asset-pricing research has evolved into a battleground of factor models (e.g., Hou, Xue, and Zhang's (2015) Q-factor model, Fama and French's (2018) six-factor model (FF6), Hou, Mo, Xue, and Zhang's (2021) Q5 model, Stambaugh and Yuan's (2017) three- (SY3) and four-factor (SY4) models, Barillas and Shanken's (2018) six-factor model (BS6), and Daniel, Hirshleifer, and Sun's (2020) three-factor model (DHS3)),[2] each striving to refine our understanding of expected return determination.

Against this backdrop, the comparison of different asset-pricing models has recently garnered critical empirical attention to determine which factors best explain expected returns. Recent studies,

[1] These anomalies primarily revolve around firm characteristics, such as size (Banz, 1981), value (Stattman, 1980), and momentum (Jegadeesh and Titman, 1993).

[2] A discussion of these models is presented in Section 2.

such as Jensen, Kelly, and Pedersen (2023) and Swade, Hanauer, Lohre, and Blitz (2024), have investigated factor replicability and alpha contribution in the U.S. market. Our research contributes to the literature by employing factor-spanning and other asset-pricing tests to provide one of the earliest pieces of Canadian-based evidence on the relative performance of the major asset-pricing models. The work of Huber, Jacobs, Müller, and Preissler (2023) is germane to our study. They evaluate the performance of competing factor-based asset-pricing models in describing the cross-section of stock returns across international regions, including North America, Europe, Asia-Pacific (excluding Japan), and Japan. However, their region-based analysis assumes that equity markets are integrated within regions but segmented between regions.

Our research endeavor holds significant value for several reasons. *First*, focusing on a single country allows for a more homogeneous sample in terms of financial and economic development, legal structure, corporate governance, and industrial structure, which may impact the relevance of factors in expected returns. *Second*, non-U.S. evidence can add value since much of the existing literature on asset-pricing model comparison focuses on U.S. market data, influenced by the prevalent U.S. home bias in academic research (Karolyi, 2016). [3] *Third*, many international asset-pricing studies overlook the need for country-specific results despite (i) the debatable validity of full market integration hypotheses (Huber et al., 2023), (ii) the higher ability of local or country-specific factors to explain time-series variation in international stock returns (Griffin, 2002; Hanauer and Linhart, 2015; Hollstein, 2022; Huber et al., 2023), and (iii) the potential for grouping data across countries to significantly affect inferences (Ang, Liu, and Schwarz, 2020). *Fourth*, Canada offers a unique setting for examining our

[3] "…only 16% (23%) of all empirical studies published in the top four (fourteen) Finance journals examine non-US markets" (Karolyi, 2016, p.2049)

research questions, given its significant global equity market capitalization (SIFMA, 2023). Additionally, the Canadian and U.S. stock markets remain segmented, characterized by distinct valuations and costs of capital (King and Segal, 2008). This underscores the importance of extending the applicability of U.S. findings to other markets with comparable institutional environments to avoid the data-snooping problem (Lo and MacKinlay, 1990; Lam and Tam, 2011).[4] This is particularly relevant given the ongoing debate surrounding the replication crisis in finance (Harvey, Liu, and Zhu, 2016; Hou, Mo, Xue, and Zhang, 2022; Chen and Zimmermann, 2022; Jensen et al., 2023). Notwithstanding the perceived economic integration between Canada and the U.S. and their shared institutional frameworks (e.g., Irvine, 2000; La Porta, Lopez-de-Silanes, and Shleifer, 2006; Bargeron, Lehn, and Zutter, 2010), substantial regulatory and corporate governance differences persist (Attig et al., 2006; Nicholls, 2006; Kryzanowski and Zhang, 2013). For example, Baker, Dutta, and Saadi (2011) demonstrate that ownership concentration is higher in Canadian firms than in U.S. firms, and the Canadian corporate governance regime is largely voluntary and perceived as weaker than its U.S. counterpart. These nuances may influence market preferences toward certain factors (Athanassakos and Ackert, 2021), leading to a valuation discount for equity of Canadian-listed firms compared to those cross-listed on Canadian and U.S. markets (King and Segal, 2008).

More broadly, the differential performance of global, regional, and local factor models cannot be solely attributed to market integration or segmentation (Huber et al., 2023; Swade et al., 2024). This suggests that investor preferences and local market characteristics play a crucial role, emphasizing the need for country-specific factor models. Empirical analysis along this line is warranted since most factors have only been proposed recently, and the jury is still out on their relevance because existing

[4] Canada and the U.S. show a high level of cultural, economic, and regulatory convergence (Doukas and Switzer, 2000; Mittoo, 2003).

findings seem to be a matter of contention (Ahmed, Bu, and Tsvetanov, 2019; Avramov et al., 2023; Jensen et al., 2023).[5]

We initiate our empirical investigation by manually matching the Canadian Financial Markets Research Center database (CFMRC-TSX) with the COMPUSTAT database and constructing 17 factors associated with 11 multifactor asset-pricing models, covering the period from July 1991 to December 2022. In our first test, we examine the redundancy of HML in describing Canadian returns (Fama and French, 2015) and find that HML exhibits a significant monthly average return of 0.88% (t = 2.70). However, when HML is regressed against its peers in the FF5 model, the resulting alpha is approximately half of the raw premium (44 basis points per month) and is statistically insignificant at the 10% level (t = 1.45). This result aligns with U.S.-based evidence that the value factor is redundant in describing returns when controlling for investment and profitability factors.

In our second test, we investigate the relative importance of the monthly updated value factor (HML_M) proposed by Asness and Frazzini (2013). HML_M is neither redundant in the BS6 model (t = 4.37) nor completely subsumed by the FF6, Q5, and SY4 models, indicating that the monthly updated value factor is relevant for pricing Canadian securities.

Our third test assesses whether the Q-factor and Q5 models subsume the momentum factor UMD. The findings reveal that the alphas obtained in the spanning Q and Q5 regressions are -0.20% and -0.24% per month, respectively, and none of these alphas is statistically significant at conventional levels. These results align with those of Hou et al. (2015) and Hou, Mo, Xue, and Zhang (2019) in the U.S. markets.

[5] For instance, Stambaugh and Yuan (2017) highlight the potency of their mispricing-based models with management- and performance-related factors. Yet, Mbengue, Ndiaye, and Sy (2023) find that these factors perform poorly in African stock markets.

When we examine whether avoiding mispriced stocks leads to the construction of a better size factor SMB_P (i.e., our fourth test), we find that SMB_P and SMB yield nearly identical average returns from 1991 to 2022 (0.35% compared with 0.36% per month). This finding contradicts Stambaugh and Yuan's (2017) U.S. market observations. The recorded average premiums for SMB_P and SMB are insignificant, consistent with a diminished post-publication size effect (e.g., Dichev, 1998; Chan, Karceski, and Lakonishok, 2000; McLean and Pontiff, 2016). However, the size factor R_{ME} proposed by Hou et al. (2015) generates a significant average premium of 0.77% per month (t = 3.69) and remains substantial in most redundancy and spanning regressions. This suggests that the choice of construction method significantly impacts the size factor estimation.

In our subsequent tests based on factor-spanning regressions, we first investigate the alleged dominance of investment-based models over the FF5 and FF6 models. Consistent with the results of Hou et al. (2019) on U.S. data, we find that the Q-factor and Q5 models subsume the FF5 and FF6 models because none of the factors in Fama and French's models generates reliable abnormal returns in the Q and Q5 spanning regressions, whereas the R_{ROE} and R_{EG} factors consistently yield significant alphas in the FF5 and FF6 spanning regressions. Next, we examine whether the Q5 model subsumes the SY4 model in factor-spanning tests and confirm Hou et al.'s (2019) U.S.-based evidence. Despite having significant average raw premiums, none of the mispricing factors (MGMT and PERF) secures a reliably positive alpha in the Q5 spanning regressions, whereas R_{ROE} and R_{EG} produce reliable positive alphas in SY4 spanning regressions. Finally, contrasting the Q5, DHS3, and BS6 models, we find that none can be spanned by its counterparts in the factor battle. For instance, the Q5 model falls short of subsuming the DHS3 and BS6 models because of its inability to explain Canada's substantial post-earnings announcement drift effect and nullify the alpha associated with HML_M. The DHS3 model does not subsume the Q5 and BS6 models due to the positive alphas retained by R_{ME}, R_{ROE}, and R_{EG}

when regressed against the DHS3 factors.[6] Despite its designation as the premier model by Barillas and Shanken (2018), BS6 fails to subsume the Q5 model due to its inability to account for the expected growth factor (REG) and the DHS3 model due to its failure to encompass the PEAD factor.

Taken together, our evidence underscores the inherent value of recently proposed models, as they introduce unique factors not subsumed by others. Yet, these models remain incomplete when considered in isolation, as they cannot comprehensively explain all alternative factors. This begs a critical query: Which model effectively prices Canadian securities? Our analysis paves the way for a six-factor pricing model (SFPM) consisting of the market, R_{ME}, HML_M, R_{ROE}, R_{EG}, and PEAD. Robustness tests confirm that none of these factors is redundant in explaining Canadian returns. When we subject the ten multifactor models, along with the SFPM, to a series of asset-pricing tests to evaluate their ability to explain anomalies in the Canadian markets, our SFPM emerges as the top-performing model. Indeed, it outperforms its counterparts on all performance metrics except for the average adjusted R-squared, where it ranks second to the BS6 model. Its statistically significant GRS test result is related to its inability to fully explain the extreme return differentials of portfolios sorted by book-to-market and accruals, but successfully leads to a diminishing profitability of strategies based on these anomalies.

The remainder of this paper is organized as follows. Section 2 provides a brief review of the relevant literature. Section 3 presents the data used in this study. Section 4 outlines the methodology for factor construction and presents summary statistics for these factors. Section 5 discusses the results of the redundancy tests. Section 6 presents the spanning test results, and Section 7 focuses on

[6] Similarly, in contrast to Daniel et al.'s (2020) finding using U.S. markets data, the behavioral factors FIN and PEAD cannot explain Fama and French's HML and UMD factors.

determining the best asset-pricing model for Canadian markets. Section 8 compares the models' abilities to explain anomalies. Finally, Section 9 concludes the paper.

2. Literature Background

In his American Finance Association Presidential Address, Cochrane (2011) termed the plethora of factors the "factor zoo." A recent study by Hou, Xue, and Zhang (2020) delved into 452 potential return predictors in the literature, revealing that 65% failed to meet the minimal threshold of replicability.[7] The identification of the most pivotal factors remains a focal point of the ongoing research debate and controversy, as highlighted by Avramov et al. (2023). Given the sheer scope of the literature, this study refrains from an exhaustive review (for that, see Martino and Puopolo, 2022). Instead, the focus is directed toward the most widely utilized factors in classic and new asset-pricing models.

Fama and French (1993) extend the CAPM by introducing a three-factor pricing model (TFPM) that includes size (SMB) and value (HML) factors. Although the rationale behind including SMB and HML is empirically grounded, Fama (1996) argues that they can be interpreted as hedge factors. Numerous studies, including those by Liew and Vassalou (2000), Hahn and Lee (2006), and Jagannathan and Wang (2007), support this perspective by showing that SMB and HML capture fundamental risk associated with the intertemporal marginal rate of substitution.[8] However, Fama and French (1996) note that TFPM's most serious challenge lies in its inability to explain the momentum effect, a shortcoming addressed by Carhart (1997) through the introduction of a four-factor pricing model (FFPM) that incorporates a momentum factor (UMD) into the TFPM.

[7] Harvey et al. (2016) compile 313 papers that put forth 316 different factors and provide rationales for various factor-pricing models. Feng, Giglio, and Xiu (2020) consider 150 tradable factors, Jensen et al. (2023) study 153 factors, and Cakici et al. (2023) rely on 145 anomalous variables.

[8] Ferson, Sarkissian, and Simin (1999) show, however, that attribute-sorted portfolios such as SMB and HML can appear to be priced risk factors even if the attributes are not related to risk.

In this context, the number of anomalies continued to grow (e.g., McLean and Pontiff, 2016; Harvey et al., 2016; Hou et al., 2020), and profitability and investment factors appear to stand out. The idea is that highly profitable stocks tend to outperform their less profitable counterparts (Novy-Marx, 2013), and high-investment stocks tend to underperform low-investment stocks (Titman, Wei, and Xie, 2004). Building on these findings, Fama and French (2015) extend the TFPM to a five-factor model (FF5) by integrating an investment factor (CMA) and a profitability factor (RMW). Their evidence reveals that HML loses its explanatory power and becomes redundant in FF5, a finding supported by other U.S.-based studies (Forbes, Igboekwu, and Mousavi, 2020; Hou et al., 2015).[9]

While Fama and French (2015) suggest that the redundancy of the value factor may be specific to their sample, Racicot and Théoret (2016) find that the value factor is not redundant for many strategies, as it embeds risk dimensions not accounted for by other factors. International evidence also supports the relevance of the value factor, with Grobys and Kolari (2022) finding that it matters in Europe, Asia (excluding Japan), and Japan through spanning regressions. Analyzing the German market, Dirkx and Peter (2020) find that not only does HML fail to generate a positive premium when considered in isolation or when challenged by its peers in the FF6 model, but also the profitability and investment factors do not add significant explanatory power. Ammann, Hemauer, and Straumann (2023) examine the controversy surrounding the redundancy of the value factor, arguing that the value and investment factors ought to be correlated by construction because they are subject to the reality behind the dividend discount model and the net present value rule. They find that the value factor is not redundant when constructed from stocks likely to provide reliable information about cash flows and expected returns.

[9] Hou et al. (2015) claim that the value factor is a noisy version of their investment factor in the Q-factor model.

This lack of consensus on the relevance of HML motivates our investigation into whether this factor is redundant in the Canadian markets.[10] Asness and Frazzini (2013) highlight the intricacies of the value factor, arguing that the standard approach to measuring HML is problematic due to its reliance on lagged asset prices. They propose using more contemporaneous price information to approximate the true, unobservable book-to-market ratio commonly employed to classify value and growth stocks, resulting in a more precise monthly updated value factor (HML_M). In contrast to previous results regarding the value factor, recent studies (Swade et al., 2024; Barillas and Shanken, 2018; Hanauer, 2020) find that HML_M is not redundant in describing the cross-sections of asset returns across various markets.

Fama and French (2018) introduce a six-factor model (FF6) by incorporating the momentum factor (UMD) proposed by Carhart (1997) into their FF5 model, albeit reluctantly, "to satisfy insistent popular demand" (p.237), as it lacks theoretical motivation despite being empirically robust (Jagadeesh and Titman, 1993). Ehsani and Linnainmaa (2022) argue that momentum is not a distinct risk factor and merely times other factors in the U.S. markets, while Cakici et al. (2023) challenge this view, revealing that the factor momentum cannot subsume the stock momentum in global markets. We contribute to this ongoing debate by investigating whether the momentum factor is subsumed in the new multifactor models that do not consider it, including the Q-factor and Q5 models (Hou et al., 2015, 2021).[11]

[10] A previous Canadian study by L'Her, Masmoudi, and Suret (2004) shows that the book-to-market factor returns are positive (negative) and highly (barely) significant in down-markets (up-markets), but the value premium is only significant in an expansive environment. Also, Athanassakos and Ackert (2021) suggest that the value premium still exists in Canada, especially for stocks with low prices.

[11] This is important because Hou et al. (2015) find a high correlation between UMD and R_{ROE} (about 50%) and report a small but statistically insignificant alpha of UMD in the Q-factor model.

Derived from the first principle of investment, the Q-factor model and its extensions focus on the supply side of investment, incorporating factors based on market, size (R_{ME}), investment ($R_{I/A}$), and profitability (R_{ROE}). Hou et al. (2021) extend the Q-factor model by adding an expected growth factor (R_{EG}), leading to the Q5 model. While Hou et al. (2019) find that the Q-factor model largely subsumes Fama–French's five- and six-factor models in factor-spanning tests based on U.S. data, Ahmed, Bu, Symeonidis, and Tsvetanov (2023) argue that models known for explaining return anomalies may not be the best for capturing systematic return covariation. Although Hou et al. (2022, p.20) recently concluded that the Q5 model does a decent job explaining "the performance of prominent quantitative security analysis strategies as well as that of best-performing active, discretionary equity funds," its performance outside the United States is yet to be explored.

Stambaugh and Yuan (2017) present two models that aggregate 11 prominent anomalies into parsimonious sets of factors. In the first model, they construct a management-based factor (MGMT) and a profitability-based factor (PERF) by clustering anomalies related to management and performance, respectively. [12] By combining MGMT and PERF with the market factor and a size factor that does not consider mispriced stocks in its construction (SMB_P), the authors obtain a mispricing-based four-factor model (SY4). They also consider a single mispricing factor (UMO) that aggregates information from all 11 anomalies, resulting in a three-factor model (SY3) when combined with the market and size factors. Stambaugh and Yuan (2017) find that the SY4 model outperforms the FF5 and Q-factor models in factor-spanning tests with U.S. data.

Daniel et al. (2020) propose a financing-based mispricing factor (FIN) designed to capture long-horizon return anomalies stemming from managers' decisions to issue or repurchase equity in response

[12] The anomalies include net stock issues, composite equity issues, accruals, net operating assets, asset growth, and investment to assets in the first cluster denoted MGMT; the second cluster, PERF, includes the other five anomalies related to performance: distress, O-score, momentum, gross profitability, and return on assets.

to persistent mispricing. They also incorporate a post-earnings announcement drift factor (PEAD) to address short-horizon return anomalies, considering that some investors fail to utilize information in earnings announcements correctly. Combining the FIN and PEAD factors with the market, they establish a three-factor pricing model (DHS3). When applied to U.S. data, their findings suggest that pricing models such as the TFPM, FFPM, FF5, and SY4 fall short of explaining the FIN and PEAD anomalies.

Barillas and Shanken (2018) determine that a six-factor model (BS6) consisting of the market, SMB, HML_{M}, UMD, $R_{I/A}$, and R_{ROE} provides the best posterior probability for U.S. stock returns from 1972 to 2015. In a comprehensive evaluation of ten asset-pricing models, including BS6, Ahmed et al. (2019) find that the SY4 model, albeit incomplete, provides the most accurate description of average excess returns, followed closely by the Q-factor model. Hou et al. (2019) conduct factor-spanning and GRS tests involving seven asset-pricing models, revealing the dominance of the Q-factor model over the FF6 model and the subsuming of SY4 by Q5.

While most studies on the battle of factors rely on U.S. data, recent research has extended this evidence to the international context. Huber et al. (2023) form international and regional factors for developed markets to evaluate the relative performance of seven multifactor models (TFPM, FF5, FF6, DHS3, SY4, Q, and Q5) along with the CAPM. They find that recently proposed models tend to perform better than the Fama–French models, but there is no clear winner due to regional variations. Hanauer (2020) constructs a set of international factors for 50 non-U.S. developed and emerging markets. In factor-spanning tests involving the CAPM, TFPM, FF5, FF6, BS6, SY4, and Q-factor models, Hanauer (2020) finds that the BS6 model outperforms other models. Jensen et al. (2023) also construct a sample of global factors that weigh country-specific factors in proportion to the country's total market capitalization, finding that the U.S. results replicate well across a broad sample of developed and emerging markets.[13] However, Hollstein (2022) finds that local factor models typically provide lower pricing errors than global factor models in explaining the cross-section of stock returns in individual countries.

In the U.K. market, assumed to be well-integrated into world markets, Fletcher (2019) finds that the standard approach to comparing asset-pricing models and the Bayesian approach proposed by Barillas and Shanken (2018) gives different results. However, the FF6 model emerges as the best descriptor of U.K. stock returns among the nine models considered. In contrast, Hanauer, Jansen, Swinkels, and Zhou (2024) find that the Q-factor model performs well in the more segmented Chinese A-share markets, but is outperformed by modified FF6 and FFPM models tailored to Chinese A-shares.[14]

Hollstein (2022) posits that the differential performance of global, regional, and local factor models cannot solely be ascribed to the degree of integration or segmentation of stock markets. Instead, investors' preferred habitats in local stock markets may explain the need for country-specific factor models. However, Huber et al. (2023) treat the U.S. and the Canadian markets as one regional market—the North American market—along with Europe, Asia Pacific ex Japan, and Japan.[15] Although their global perspective adds value for the generalizability of insights previously gained from the U.S. studies, it also reveals considerable model performance heterogeneity depending on the region. Blending the Canadian market (which accounts for around 3% of the world market) with the world's largest

[13] In contrast to our study, which evaluates the relative performance of ten competing factor-based asset-pricing models in Canada, Jensen et al. (2023) focus on the replicability of factors. They didn't directly evaluate the performance of a specific asset-pricing model, but provided data on all the 153 factors, clustered into 13 themes, for 93 countries.

[14] Using intraday stock returns from Chinese A-share markets, Ye, Jiang, and Luo (2023) find that the factors in the FF5 model earns reliable beta premiums, albeit at varying prices overnight versus intraday.

[15] Except for Japan, Huber et al. (2023) do not focus on any specific country. They assume regional integration and provide no information on the relative performance of the factor-pricing models in each region. However, their results reveal a tendency toward market segmentation.

market into the North American region doesn't bring out the specifics of the Canadian market since all the factors and the breakpoints of the portfolio sorts may be dominated by the U.S. data.

While both Canada and the U.S. have rigorous accounting standards and robust trading mechanisms, the specific rules and regulations vary. Furthermore, in contrast to the U.S., the stock market is highly concentrated in a few sectors in Canada, with the financials, energy, and materials sectors representing a large portion of the market. Differences in accounting rules, trading mechanisms, industry structure, ownership structure, and corporate governance add to the need to investigate the relative performance of factor-based asset-pricing models in the Canadian markets.

To the best of our knowledge and the literature at hand, this study stands as the first to explore the battle of factors in the Canadian context. This academic endeavor is relevant because Barillas and Shanken (2018), Fama and French (2018), and Forbes et al. (2020), among others, emphasize the importance of comprehending the optimal combination of relevant factors, advocating for the consideration of the right group of asset-pricing factors rather than a continuous addition of factors, especially those that may prove redundant.

3. Data

We start with the list of stocks available on the Canadian Financial Markets Research Center database (CFMRC-TSX) and focus on TSX-listed ordinary common stocks, excluding other instruments such as REITs, income trusts, and exchangeable shares. All our tests are based on monthly returns. For firms with multiple class shares, we consolidate the market capitalizations of the distinct classes and assign them to the class boasting the largest market capitalization. We then manually match TSX firms with COMPUSTAT yearly and quarterly files, refining our sample to encompass only entities with COMPUSTAT financial data. We then exclude firms with negative book values, financials (SIC codes 6000–6999), utilities (SIC codes 4900–4999), and non-operating establishments (SIC codes 9000–9999). These filters yield a sample of 192,533 observations, representing 1,834 stocks.[16]

To determine the study period, we examined the temporal progression of the sample. Although data were available from January 1980, the number of stocks was limited (only seven). The coverage expanded to 38 stocks in July 1984, but it was not until April 1991 that the number of available stocks reached a substantial level (247). In line with this data availability, our sample period commences in July 1991, ensuring a consistent and comprehensive dataset (1,810 stocks and 183,925 observations).

4. Factor Construction and Performance

This study focuses on factors extracted from ten asset-pricing models. The first four we consider are Fama and French's (1993) TFPM and its extensions, as delineated below:

$$E[R_i] - R_F = b_i(E[R_M] - R_F) + s_i E[SMB] + h_i E[HML], \tag{1}$$

$$E[R_i] - R_F = b_i(E[R_M] - R_F) + s_i E[SMB] + h_i E[HML] + u_i E[UMD], \tag{2}$$

$$\begin{aligned} E[R_i] - R_F &= b_i(E[R_M] - R_F) + s_i E[SMB] + h_i E[HML] \\ &+ r_i E[RMW] + c_i E[CMA], \end{aligned} \tag{3}$$

$$\begin{aligned} E[R_i] - R_F &= b_i(E[R_M] - R_F) + s_i E[SMB] + h_i E[HML] \\ &+ u_i E[UMD] + r_i E[RMW] + c_i E[CMA]. \end{aligned} \tag{4}$$

[16] Employing Canadian stock market data offers the advantage of sourcing accounting data from COMPUSTAT, and the definitions of characteristics employed for factor construction are harmonized with those used in U.S. markets.

Equations (1) and (2) portray the TFPM and FFPM, respectively, featuring the market ($R_M - R_F$), size (SMB), value (HML), and momentum (UMD) factors. Extending these, (3) and (4) represent the FF5 and FF6 models, introducing the investment factor (CMA) and profitability factor (RMW).

Next, we consider the Q-factor and Q5 models, illustrated in (5) and (6):

$$E[R_i] - R_F = \beta_M^i(E[R_M] - R_F) + \beta_{ME}^i E[R_{ME}] + \beta_{I/A}^i E[R_{I/A}] + \beta_{ROE}^i E[R_{ROE}], \tag{5}$$

$$E[R_i] - R_F = \beta_M^i(E[R_M] - R_F) + \beta_{ME}^i E[R_{ME}] + \beta_{I/A}^i E[R_{I/A}]$$

$$+\beta_{ROE}^i E[R_{ROE}] + \beta_{EG}^i E[R_{EG}]. \tag{6}$$

These models, grounded in the supply-side equilibrium, add size (R_{ME}), investment ($R_{I/A}$), profitability (R_{ROE}), and expected growth (R_{EG}) to the market factor.

Finally, we consider these four recently proposed mispricing-related models:

$$E[R_i] - R_F = \beta_M^i(E[R_M] - R_F) + \beta_S^i E[SMB_P] + \beta_U^i E[\mathrm{UMO}], \tag{7}$$

$$E[R_i] - R_F = \beta_M^i(E[R_M] - R_F) + \beta_S^i E[SMB_P]$$

$$+\beta_{MGMT}^i E[MGMT] + \beta_{PERF}^i E[PERF], \tag{8}$$

$$E[R_i] - R_F = \beta_M^i(E[R_M] - R_F) + \beta_F^i E[FIN] + \beta_P^i E[\mathrm{PEAD}], \tag{9}$$

$$E[R_i] - R_F = \beta_M^i(E[R_M] - R_F) + s_i E[SMB] + h_i E[HML_M] + u_i E[UMD]$$

$$+\beta_{I/A}^i E\left[R_{I/A}\right] + \beta_{ROE}^i E[R_{ROE}]. \tag{10}$$

Equations (7) and (8) depict the mispricing-based models SY3 and SY4, aggregating information across large clusters of anomalies. Specifically, (7) aggregates information from 11 anomalies through the UMO factor, while (8) combines six management-related anomalies through MGMT and five performance-related anomalies through PERF. Equation (9) embodies the DHS3 model, considering market, financing-based mispricing (FIN), and post-earnings announcement drift (PEAD) factors. Finally, (10) outlines the BS6 model, blending market, SMB, UMD, $R_{I/A}$, R_{ROE}, and HML_M factors.

All pricing models examined in this study incorporate a core market factor, represented by the value-weighted return of all Canadian stocks minus the monthly return on the 91-day Canadian Treasury bill. To derive the size and value factors associated with the TFPM, we adhere to the methodology outlined by Fama and French (1993). Employing this approach, we create six value-weighted portfolios at the end of each preceding June.[17] These portfolios result from intersecting two size groups (using median market capitalization breakpoint) and three book-to-market groups (determined by the 30% and 70% book-to-market breakpoints). The SMB factor captures the differential average return between the three small and three large portfolios, while HML captures the difference in average returns between the two value (high book-to-market) and two growth portfolios.[18] Similarly, we construct the momentum factor using six portfolios formed by intersecting two size groups and three prior (2–12)-month returns groups. UMD is the average return differential between the two winning and two losing portfolios.

We adopt the methodology Fama and French (2015) introduced to measure profitability and investment factors. Following the usual approach, we generate six portfolios by intersecting two size groups and three operating profitability (OP) groups. OP is computed as the disparity between revenues and costs divided by book equity. RMW represents the average return differential between the two

[17] Following Fama and French (1992), we always match stock returns from July of year *t* to June of *t*+1 to firm characteristics for the fiscal year ending in *t*–1 to ensure that only available information is used to construct the factors. We update the portfolios every June and hold their composition constant for a year.

[18] Throughout this study, for the sake of simplicity, we tabulate only the results on the classical SMB factor obtained using the six size-value portfolios. The SMB factors obtained from (i) six size-profitability portfolios, (ii) six size-investment portfolios, (iii) and 18 portfolios (six size-value portfolios, six size-investment portfolios, and six size-profitability portfolios) yield similar results.

portfolios with robust operating profitability and the two portfolios with weak operating profitability. Similarly, CMA captures the average return differential between the two portfolios with conservative investment and the two portfolios with aggressive investment.

We follow Hou et al. (2015) as much as possible in crafting the Canadian counterparts of the factors associated with the Q-based models. To this end, we construct the size, investment, and profitability factors using 18 portfolios from the intersections of two size, three investments-to-assets (I/A), and three ROE groups. The size factor (R_{ME}) is calculated as the average return differential between the nine small and nine big portfolios. The investment factor ($R_{I/A}$) is the average return differential between the six low and six high I/A portfolios. The profitability factor (R_{ROE}) is the average return differential between the six high and six low ROE portfolios. Adhering to the methodology articulated by Hou et al. (2019, 2021), we construct the expected growth factor by creating six portfolios through the intersection of two size groups and three groups based on expected investment-to-assets growth, denoted as E[△I/A]. Mirroring the authors' approach, we gauge E[△I/A] through ten-year rolling predictive regressions, employing one-year investment changes as the regressand. We utilize the logarithm of Tobin's Q, operating cash flow scaled by total assets, and one-year change in ROE as regressors. The expected growth factor (R_{EG}) represents the average return differential between the two portfolios with high E[△I/A] and the two portfolios with low E[△I/A].

Following Stambaugh and Yuan (2017), we formulate two composite mispricing measures. The first measure (P1) combines six anomalies related to management, including net stock issues, composite equity issues, accruals, net operating assets, asset growth, and investment-to-assets. The second measure (P2) combines four performance-related anomalies: O-score, momentum, gross profitability, and return on assets.[19] To compute SMB_P, we avoid relying on stocks in the extreme mispricing categories and instead focus on stocks in the middle groups of mispricing sorts. Utilizing the P1 and P2, we form two sets of 2×3 sorts based on size and mispricing measures, thereby generating 12 distinct portfolios. In line with Stambaugh and Yuan's (2017) approach, our methodology deviates from the customary 30% and 70% breakpoints when crafting the MGMT, PERF, and UMO factors. Instead, we use the 20% and 80% breakpoints to form these factors. SMB_P represents the average return differential between the two portfolios in the middle mispricing groups categorized by small and large size. To compute MGMT, we initially distributed stocks into six portfolios by intersecting two size groups and three P1 groups. MGMT captures the average return differential between the portfolios with high P1 and low P1. Similarly, we construct PERF using P2 instead of P1. Finally, we aggregate the information from the 11 anomalies into a single mispricing factor known as UMO (Underpriced minus Overpriced).

We adhere to the methodology outlined by Daniel et al. (2020) to construct the financing-based mispricing factor (FIN) utilizing the 1-year net share issuance (NSI) and 5-year composite share issuance (CSI). Beginning with the customary end-of-June starting point, we establish two size groups and three financing groups based on NSI and CSI. These financing groups are determined using the specific rankings detailed by the authors, particularly for sorting stocks into NSI groups. Subsequently, we form six portfolios by intersecting the two size groups with the three financing groups. The FIN factor is computed as the average return difference between the portfolios with low financing and those with high financing. To construct the PEAD factor, we quantify the earnings surprise by utilizing the 4-day cumulative abnormal return (CAR) surrounding the most recent quarterly earnings announcement date,

[19] When constructing the Canadian version of the P2 measure, we chose to remove the distress characteristic because this variable has many missing values and brings a lot of noise.

obtained from the COMPUSTAT quarterly item RDQ. By forming the customary six portfolios based on the intersection of two size groups and six CAR groups, the post-earnings announcement drift factor (PEAD) is computed as the average return difference between the portfolios with high earnings surprise and those with low earnings surprise.

Finally, we adopt the approach of Asness and Frazzini (2013) to construct the monthly updated value factor, HML_M. This factor hinges on book-to-market, using the most recent monthly stock price in the denominator.

Table 1, Panel A, presents summary statistics for the factors from July 1991 to December 2022. Canada's average excess market return stands at 0.53% per month (or 6.36% per annum), demonstrating statistical significance at the 5% level. Of the 17 examined factors, only four do not display statistically significant average monthly returns. These factors include SMB (0.36%, t = 1.51), SMB_P (0.35%, t = 1.41), RMW (-0.06%, t = -0.25), and FIN (-0.11%, t = -0.43). The nonsignificant result for FIN suggests the absence of persistent overconfidence-driven mispricing, a phenomenon often observed in the United States but not evidently present in Canadian markets. The similarity of the average premiums generated by SMB and SMB_P, along with their lack of statistical significance, is somewhat surprising, considering Stambaugh and Yuan's (2017) argument that a more potent size factor can be constructed by avoiding stocks that are most likely to be mispriced. However, this finding aligns with the well-documented diminishing post-publication impact of the size effect noted by Dichev (1998), Chan et al. (2000), and McLean and Pontiff (2016), among others. Intriguingly, the size factor R_{ME}, constructed using Hou et al.'s (2015) approach—which relies on 18 portfolios sorted by size, I/A, and ROE—produces a reliable average premium of 0.77% per month (t = 3.69), suggesting that SMB is sensitive to the characteristics considered in its construction.

TABLE 1 ABOUT HERE

In line with the well-recognized value and momentum effects, HML and UMD generate average monthly premiums of 0.88% and 1.08%, respectively, maintaining statistical significance at the 1% level. The monthly updated value factor HML_M is also robust, with a monthly premium of 0.93% (t = 2.55). Adding to the insights from Hou et al. (2019) in the U.S. markets, the expected growth factor R_{EG} emerges as a significant player, displaying an average monthly return of 2.06% (t = 4.37).

Consistent with U.S. findings (Fama and French, 2015; Hou et al., 2015) but diverging from results in some emerging markets [see, e.g., Foye and Valentinčič (2020) for the Indonesian market and Mbengue et al. (2023) for 13 African markets], the two investment-based factors inherent in the FF5 and Q5 models appear priced in Canada. The average value of CMA for 1991–2022 stands at 61 basis points per month, while that of $R_{I/A}$ is even higher at 91 basis points; both figures remain highly statistically significant with t-statistics over three.

Yet, some of the standout performers during the examined period exhibit behavioral attributes. While MGMT delivers an average monthly return of 1.61% (t = 5.31), UMO generates 2.08% (t = 5.75), and PEAD truly stands out with an impressive average monthly return of 6.82% (t = 20.34). The latter result suggests that short-horizon earnings surprise is potentially a key factor influencing Canadian equity performance.

Panel B of Table 1 displays the pairwise correlation coefficients among the factors. While most correlations are either negative or small, implying potential risk management benefits through factor diversification (Nazaire, Pacurar, and Sy, 2021), some factors exhibit strong associations, with correlation coefficients surpassing 50% in absolute terms. Notably, the two value factors considered, HML and HML_M, demonstrate a high correlation of approximately 57%, begging the question of whether the monthly updating of HML, as proposed by Asness and Frazzini (2013), bolsters the effective pricing of securities. Offering a possible explanation for why the value factor is often redundant in the FF5 and Q-factor models, HML shows a high correlation with CMA and $R_{I/A}$, with correlation coefficients exceeding 42%. The R_{ROE}, PERF, and UMO factors reveal pronounced correlations, suggesting their measurement of a common underlying phenomenon. Additionally, these profitability factors display notable associations with the momentum factor, potentially shedding light on the efficacy of the investment-based (Q-factor and Q5) and mispricing-based (SY3 and SY4) models in capturing momentum effects.

Finally, it is noteworthy that the mispricing factors FIN and PEAD recently introduced by Daniel et al. (2020) exhibit relatively modest correlations with the remaining 15 factors. Particularly, PEAD demonstrates its highest correlation with CMA, even though this correlation stands at just 20%. This finding, coupled with the substantial average realization of the factor, implies that PEAD is unlikely to be overshadowed or subsumed by other factors.

5. Factor Redundancy Tests

Redundancy tests assess whether a factor is encompassed by its counterparts within a specific asset-pricing model. Fama and French (2015) conduct redundancy tests on the factors in the FF5 model, primarily motivated by the disparity between this model and its close competitor, the Q-factor model, regarding the inclusion of the HML factor. While their empirical analysis finds HML redundant in describing returns, the authors cautiously acknowledge that this result might only hold for U.S. data from 1963 to 2013. Subsequent studies investigating the redundancy of HML have yielded conflicting results. Leite et al. (2018) find the value factor redundant in describing returns in the emerging markets from three regions (Asia, Eastern Europe, and Latin America), while in contrast, Mbengue et al. (2023) point out that HML is not redundant in the more-frontier African markets. Additionally, Barillas and Shanken (2018) observe that the monthly updated HML factor is neither redundant nor subsumed by other asset-pricing models. These divergent findings highlight the potential contextual nature of factor redundancy, thus underscoring the importance of conducting a comprehensive analysis across different markets.

Table 2 provides the results of the redundancy tests, focusing on the ability of each factor to maintain a positive alpha when regressed on its peers in the same asset-pricing model. Columns 3 to 12 present the results for each of the ten models examined. The first row of the table examines the redundancy of the market factor across all models. For all ten models except DHS3, the redundancy of the market factor is rejected at the 5% level, confirming its importance for pricing Canadian securities. Even in the case of the DHS3 redundancy regression, the market factor still exhibits an alpha close to its average value (compare 0.53% to 0.45% per month). These findings emphasize the importance of the market factor as a key driver of asset returns in Canadian capital markets.

TABLE 2 ABOUT HERE

Consistent with the findings from the U.S. data, the HML factor proves redundant in the FF5 model. The regression of HML against the four other factors yields an insignificant alpha of 44 basis points (t = 1.45). This insignificance persists in the FF6 redundancy regression, registering a t-statistic of only 1.31. As discussed in Section 2, the redundancy of HML exists because HML and CMA ought to be correlated by construction (Ammann et al., 2023). Nonetheless, the monthly updated HML factor retains its significance in the BS6 model, exhibiting a robust alpha of 1.47% per month (t = 4.37).

For the classical size factor SMB, none of its alphas in the redundancy tests conducted across the TFPM, FFPM, FF5, and FF6 models exhibits reliably positive estimates at the standard significance levels. This lack of significance can be attributed to SMB's inability to generate a reliable average return over the analyzed sample period. The mispricing-purged size factor proposed by Stambaugh and Yuan (SMB_P) passes (at the 5% level) the redundancy test in the SY4 regression but fails in the

SY3 regression. The R_{ME} factor is not redundant in the Q-factor and Q5 regressions, with corresponding *t*-statistics of 2.54 and 2.77, respectively.

The investment factor $R_{I/A}$ exhibits reliable positive alphas on the Q and Q5 redundancy regressions with respective *t*-statistics of 2.50 and 2.46. However, it cannot produce a significant alpha (at the 10% level) when regressed on peers in the BS6 model. On the other hand, while CMA does not show redundancy in the FF5 model (t = 2.89), the alpha obtained when regressed on its peers from the FF6 model is only significant at the 10% level (t = 1.91).

While the momentum factor produces highly significant alphas in the FFPM and FF6 redundancy regressions, with *t*-statistics exceeding 3.5, it generates a reliably positive alpha only at the 10% level (t = 1.86) when regressed on its peers from the BS6 model. This near redundancy can be attributed to the high correlation (57.49%) between UMD and R_{ROE}, which is consistent with the findings of Hou et al. (2019) in the United States.

Both the profitability factor R_{ROE} and the expected growth factor R_{EG} exhibit non-redundancy in the Q-factor and Q5 models. However, the most notable result in Table 2 is that none of the four significant mispricing or behavioral factors (UMO, MGMT, PERF, and PEAD) demonstrates redundancy when subjected to the scrutiny of its respective peers.

6. Factor-Spanning Tests

In the pursuit of identifying superior asset-pricing models, researchers grapple with challenges such as measurement errors in factor loadings (Miller and Scholes, 1972) and sensitivity to the selection of test assets (Lewellen, Nagel, and Shanken, 2010). Moreover, multivariate tests such as the Gibbons–Ross–Shanken (1989) *F*-test (GRS test) often reject asset-pricing models when evaluated using characteristic-sorted test assets. To overcome these limitations, Fama and French (2015, 2018), Stambaugh and Yuan (2017), Hou et al. (2019), among others, have adopted factor-spanning tests (Huberman and Kandel, 1987) to compare and assess various asset-pricing models.

Factor-spanning tests involve regressing the factors of a given asset-pricing model on those of a benchmark asset-pricing model. If the model under evaluation is correct, at least one of its factors should exhibit a significant alpha in the spanning regressions. Conversely, if the benchmark model spans the pricing model under investigation, none of the assessed factors should demonstrate a reliably positive alpha in the spanning regressions. By focusing on the models' abilities to explain each other's factors, the spanning approach offers a key advantage by avoiding the need for arbitrary test assets. As Barillas and Shanken (2018) demonstrate, "all that matters when comparing two asset-pricing models is the extent to which each model prices the factors in the other model" (p.739). This simplicity and the focus on pricing factors rather than arbitrary test assets make the spanning approach an effective tool for model evaluation.

We present the spanning test results in Table 3. The table is structured as follows: the first column lists the 17 examined factors, while the second column reproduces their average returns, as presented in Table 1. The remaining ten columns correspond to the benchmark asset-pricing models considered. In this 17×10 matrix, we report the spanning regression alphas of each factor relative to the benchmark asset-pricing models.

TABLE 3 ABOUT HERE

In their comparative analysis of the FFPM and Q-factor models, Hou et al. (2019) find that the alphas of HML and UMD in the Q-factor regressions are small and insignificant, hinting at the possibility that these factors might be noisy versions of the Q-factors in U.S. markets. This result applies to Canadian markets despite differences between markets and sample periods, as the Q-factor and Q5 models comprehensively account for all factors in the FF5 and FF6 models. None of the SMB, HML, CMA, RMW, or UMD factors shows a reliably positive alpha in the Q and Q5 spanning regressions. Despite its substantial average premium of 0.88% per month (t = 2.70), HML can produce an alpha of only -0.12% (t = -0.12) in the Q regression and 0.07% (t = 0.07) in the Q5 regression. This result is explained by the strong correlation between HML and $R_{I/A}$ of approximately 43%. The spanning of CMA by the investment-based models is also attributed to its high correlation with $R_{I/A}$ (about 79%). In contrast, the Q-factor and Q5 models can explain the momentum factor because prior winning (losing) stocks tend to be those that have generated the highest (lowest) ROE, to the extent that UMD is highly correlated (57.49%) with R_{ROE}.

Similar to investment-based models, the mispricing-based SY3 and SY4 models fully encompass the factors in the FF5 and FF6 models. This is because none of SMB, HML, CMA, RMW, and UMD remains viable, even at the 10% significance level, when challenged by the factors from the SY3 or SY4 models in spanning regressions. This result aligns with Stambaugh and Yuan's (2017) findings on U.S. data. The subsuming of SMB is explained by its correlation with SMB_P (about 75%), whereas that of HML and CMA is due to their high communalities with MGMT and UMO. The latter result is not surprising given the noted value-investment relation and the fact that both MGMT and UMO consider investment as one of the aggregated anomalies. Furthermore, the mispricing-based models SY3 and SY4 can subsume the momentum factor, mainly because UMD is highly correlated (over 60%) with PERF and UMO. However, we cannot confirm in the Canadian context the findings of Daniel et al. (2020) that the behavioral factors FIN and PEAD alone can fully explain Fama and French's (2015) factors. This is because HML and UMD earn significant alphas of more than 1% per month in the DHS3 spanning regressions.

Importantly, the spanning regressions involving FF5 and FF6 show that the Fama–French factors do not fully explain the abnormal returns associated with investment-based models. For example, the FF6 alphas for the R_{ROE} and R_{EG} factors are 1.86% and 1.96% per month, respectively, with *t*-statistics of 7.25 and 4.39. The models also fail to explain the UMO, MGMT, and PERF factors associated with the SY3 and SY4 models, given that the FF5 and FF6 alphas associated with these three factors consistently exceed 60 basis points per month, with *t*-statistics over three. Based on these results, we conclude that the Q, Q5, SY3, and SY4 models subsume the FF5 and FF6 models.

When contrasting the Q-factor and Q5 models with the SY3 and SY4 models, our findings point to the domination of the investment-based models over the mispricing-based models. Specifically, we observe that both R_{ROE} and R_{EG} exhibit abnormally high returns in the spanning regressions involving SY3 and SY4, while none of the SY3 and SY4 alphas associated with SMB_P, MGMT, PERF, or UMO is reliable at the 10% level. Given this evidence, we conclude that the Q-factor and Q5 models subsume the SY3 and SY4 models in Canadian markets.

Still, none of the Q5, DHS3, and BS6 models emerges as dominant in the factors battle. The Q5 model falls short of subsuming the DHS3 and BS6 models because the Q5 alphas associated with PEAD and HML_M are reliable at the 1% level. Similarly, the DHS3 model does not subsume the Q5 and BS6 models because of the positive alphas retained by R_{ME}, R_{ROE}, and R_{EG} in the DHS3 spanning regressions. In contrast to the findings of Barillas and Shanken (2018) in the United States, where their six factors provided the best posterior probability for explaining investment returns, our Canadian-based spanning tests reveal that the BS6 model falls short of subsuming the Q5 and DHS3 models. Specifically, we find that the BS6 model cannot explain the expected growth and PEAD factors, highlighting its limitations in fully explaining the behavior of Canadian stock returns. The result on PEAD corroborates Daniel et al.'s (2020) finding that this factor offers abnormally high returns, even after adjusting for all other factors from the alternative asset-pricing models. However, the FIN does not bode as well in Canada as in the American markets.

Table 4 formally examines the factors unique to each asset-pricing model, focusing on their collective ability to produce nonzero alphas with respect to a benchmark asset-pricing model. Our analysis centers on the most recent and comprehensive models, namely FF6, Q5, SY4, DHS3, and BS6. These results reinforce our main findings, confirming that the Q5 model subsumes the FF6 and SY4 models. Indeed, in the Q5 regression, the null hypothesis that all alphas associated with HML, RMW, CMA, and UMD are jointly zero cannot be rejected, as indicated by a GRS F-statistic of 0.77 (p = 0.54). Similarly, the same test does not reject (at the 5% level) the null hypothesis that the alphas associated with MGMT and PERF (from the SY4 model) in the Q5 spanning regressions are jointly zero (GRS F = 2.39; p = 0.09). The GRS tests further validate the subsuming of the FF6 model by the SY4 model, given that the null hypothesis that the alphas of HML, RMW, CMA, and UMD in the SY4 regression are zero cannot be rejected at the standard levels of statistical significance (GRS F = 1.91; p = 0.11).

TABLE 4 ABOUT HERE

Apart from these three cases, the GRS tests suggest that no single model dominates the others in explaining return behavior in Canada. This is evidenced by all p-values associated with the GRS test being statistically significant at the 5% level. Amidst all the test results, the DHS3 model proposed by Daniel et al. (2020) stands out, producing the highest F statistic when jointly regressed on the factors of a given benchmark model. For instance, when regressed on the six factors of the FF6 model, the FIN and PEAD factors from the DHS3 model achieve an impressive F statistic of 188.66 in the GRS test. Furthermore, FIN and PEAD produce F statistics of at least 173 in the GRS tests relative to the other models. The next best-performing model (BS6) gets an F-statistic over four times lower in magnitude (38.76).

These multivariate test results underscore the importance of considering diverse factors from various asset-pricing models. Consequently, further research is warranted to delve deeper into the dynamics and interactions of these factors, shedding additional light on the best model for pricing asset returns.

7. Best Factor Model in Canada

Our quest to identify the most effective asset-pricing model has revealed that no single model dominates all others. Therefore, a more suitable approach may involve combining a parsimonious set of factors that collectively best explain returns. This section aims to identify such a model for the Canadian markets.

To initiate this process, we proceed with elimination based on our previous analysis. Four factors (SMB, SMB_P, RMW, and FIN) with limited significance are excluded due to their lack of reliably positive average returns during 1991–2022. The classical value factor HML is also removed because it is redundant and subsumed by other factors in various models. Similarly, the momentum factor UMD ought to be excluded from our quest, as it is found to be subsumed in factor-spanning regressions, including those associated with the investment-based and mispricing-based models. Additionally, the two investment-based factors, CMA and $R_{I/A}$, are dropped because they are subsumed in factor-spanning tests involving most of the remaining factors, such as those related to Q-factor, Q5, SY3, SY4, DHS3, and BS6.

After carefully eliminating factors that lack robustness or are subsumed by peers, we are left with nine potential determinants of returns. These factors include the market factor, which serves as a foundational pillar in most asset-pricing models. Additionally, the R_{ME} factor, as constructed by Hou et al. (2015), remains a significant contributor to return variation when considered alone and is not subsumed in all factor-spanning tests, except the one involving BS6.[20] We also consider three factors (MGMT, PERF, and UMO) derived from the mispricing-based models proposed by Stambaugh and Yuan (2017). These factors aggregate information across various anomalies, providing potential insights into market dynamics. Moreover, we retain the monthly updated value factor (HML_M) proposed by Asness and Frazzini (2013), which complements the traditional HML factor by incorporating more current pricing information into its construction.[21] Further, we recognize the importance of the profitability (R_{ROE}) and expected growth (R_{EG}) factors inherent in the supply-side models proposed by Hou et al. (2015) and Hou et al. (2019), respectively, shedding light on firms' operational performance and their growth potential. Finally, we acknowledge the significance of the PEAD factor, which plays a focal role in the behavioral model proposed by Daniel et al. (2020). It captures the impact of mispricing from earnings announcements, reflecting defects in investor reactions to new information. Together, these nine factors represent a comprehensive set of potential determinants that contribute to explaining the behavior of returns. By encompassing various aspects of asset pricing, from market fundamentals to investor sentiment and behavioral biases, this refined selection provides a valuable foundation for constructing a more comprehensive asset-pricing model, particularly in light of the insights gleaned from the Canadian data.

We can further narrow this list of factors for three primary reasons. *First*, the UMO factor and the combination of MGMT and PERF aggregate similar information, potentially creating redundancy. To address this, we perform the following time-series regressions:

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[20] In this BS6 regression, the subsuming of R_{ME} is due to the presence of SMB, which has a correlation of about 80% with R_{ME} (see Panel B of Table 1).

[21] HML_M produces a reliable return premium on average and is not redundant in the BS6 model. However, this value factor appears subsumed in the factor-spanning tests involving the TFPM, FF5, and DHS3. Given the high correlation between HML and HML_M (around 57%, in Panel B of Table 1), it is not surprising that the HML_M factor is subsumed in the FFPM and FF5 regressions. The subsuming of the monthly updated value factor in the DHS3 regression is due to its reliable association with the short-term behavioral factor FIN, which is not considered in our quest for the best model for the Canadian markets.

$$MGMT_t = \underset{(0.90)}{0.19} + \underset{(17.54)}{0.68} UMO_t + e \qquad R^2 = 61.55\%, \tag{11}$$

$$PERF_t = \underset{(1.49)}{0.43} + \underset{(15.26)}{0.82} UMO_t + e \qquad R^2 = 53.65\%, \tag{12}$$

$$UMO_t = \underset{(0.33)}{0.05} + \underset{(11.13)}{0.67} MGMT_t + \underset{(9.88)}{0.44} PERF_t + e \qquad R^2 = 81.98\%. \tag{13}$$

Upon analyzing the regression results (11) to (13), it is evident that UMO and the combination of MGMT and PERF measure essentially the same phenomenon. The return premium generated by UMO is a mere five basis points per month and becomes statistically unreliable at the 10% level (t = 0.33) when regressed on both MGMT and PERF. Conversely, when MGMT and PERF are regressed on UMO, the alphas obtained are statistically indistinguishable from zero at the 10% level (t = 1.04), although economically much more substantial than five basis points. Consequently, we further drop UMO and keep MGMT and PERF.

Second, we exclude MGMT from further consideration due to its subsumption in factor-spanning regressions involving Q, Q5, DHS3, and BS6.

Third, it is worth noting that R_{ROE} and PERF, which measure profitability, are highly correlated (71%). To determine which factor best captures the profitability premium, we perform the following time-series regressions:

$$R_{ROEt} = \underset{(4.96)}{1.13} + \underset{(10.38)}{0.55} PERF_t + e \qquad R^2 = 50.60\%, \tag{14}$$

$$PERF_t = \underset{(0.04)}{0.01} + \underset{(14.64)}{0.92} R_{ROEt} + e \qquad R^2 = 50.60\%. \tag{15}$$

The results of regressions (14) and (15) suggest that R_{ROE} subsumes PERF. Specifically, R_{ROE} generates a significant alpha of 1.13% per month when regressed on PERF. In contrast, PERF produces an insignificant one-basis-point abnormal performance when regressed on R_{ROE}. Accordingly, we exclude PERF from further consideration, as R_{ROE} demonstrates greater robustness and influence in capturing profitability, likely attributable to its utilization of quarterly data, which allows for a more granular incorporation of performance fluctuations over time.

This process yields a six-factor asset-pricing model (SFPM). The first factor is the market, which is a fundamental component of all asset-pricing models. Three factors—R_{ME}, R_{ROE}, and R_{EG}—originate from the investment-based models proposed by Hou et al. (2015, 2019). The remaining factors are the monthly updated value factor HML_M, introduced by Asness and Frazzini (2013) and considered by Barillas and Shanken (2018) in their BS6 model, and the post-earnings announcement drift factor PEAD, recently proposed by Daniel et al. (2020).

Table 5 presents the results of the redundancy tests, where each factor is regressed on its peers in the proposed composite model. The aim is to examine whether any of the factors are redundant in explaining Canadian returns. The reliably positive alphas reported in the second column of the table indicate that none of the six factors is redundant, suggesting that each factor plays a crucial and distinct role in explaining asset returns in Canada.

TABLE 5 ABOUT HERE

8. Which Model Best Explain Return Anomalies?

So far, we have evaluated multifactor asset-pricing models using relative tests. In this section, we conduct absolute assessments of various asset-pricing models by testing their ability to explain return anomalies. We focus on 17 anomalous premiums that form the basis of the factors used in this study.[22] They include five firm characteristics related to size, book-to-market, momentum, operating profitability, and asset growth, which are used to construct the SMB, HML, UMD, RMW, and CMA factors. Additionally, we consider the return on equity (ROE) and expected growth (E[△I/A]) characteristics used to construct the R_{ROE} and R_{EG} factors in the Q5 model, the monthly updated book-to-market characteristic used by Asness and Frazzini (2013) to construct the HML_M factor, and the composite stock issue and 4-day CAR used to construct the FIN and PEAD factors. Furthermore, we examine seven anomalous characteristics from the SY4 model, including net stock issues, composite equity issues, accruals, net operating assets, investment-to-assets, return on assets, and O-score. We compute the value-weighted returns of the bottom and top deciles for each characteristic and form the zero-investment return anomaly by taking their difference.

We assess 11 multifactor models: the TFPM, FFPM, FF5, FF6, Q, Q5, SY3, SY4, DHS3, BS6, and SFPM. Our approach involves running a system of 17 time-series regressions of return anomalies on the factors for each model. We present standard performance metrics to evaluate each model's effectiveness. Specifically, we consider seven metrics: the number of significant intercept coefficients (alphas) at the 5% level (#SIG), the average absolute monthly alphas ($A|\alpha_i|$) in percent, the average absolute *t*-values ($A|t|$), the average absolute alphas over the average absolute value of $\bar{r}_i$ (the average return on the spread portfolio i),[23] the average squared alpha over the average squared $\bar{r}_i$, the average regression R^2 (adjusted for degrees of freedom), and the *F*-statistic for the GRS test (with the associated *p*-value, which assesses whether all 17 estimated alphas are jointly zero).

[22] Since the selection of test assets can be somewhat arbitrary, we follow the approach of Stambaugh and Yuan (2017), aiming to align closely with existing literature by focusing on anomalies that researchers have deemed significant enough to incorporate into factor construction.

[23] Since the test assets considered here are zero-investment portfolios, we do not subtract the value-weighted market return as it is typically done for characteristic-sorted portfolios.

Table 6 presents the test results. Consistent with findings in the literature (see, e.g., Fama and French, 2015; Hou et al., 2015; Stambaugh and Yuan, 2016), the *p*-values associated with the GRS tests are uniformly zero to three decimal places, indicating that none of the models fully explains the 17 anomalies considered. Given these results, the common approach is to compare the models informally by examining the values obtained across various metrics. In line with our earlier result, the SFPM outperforms its counterparts on all metrics except for the average adjusted R-squared, ranking second to the BS6 model (compare an R-square of 22.60% to 22.92%). The statistically significant GRS test results for SFPM can be attributed to its inability to fully explain the extreme return differentials of portfolios sorted by book-to-market (t = -2.89) and accruals (t = -2.89). However, the negative alphas suggest the model's success in diminishing the potential profitability of strategies based on these anomalies rather than indicating market inefficiency. The SFPM model produces the smallest point estimate for the GRS F-statistic (about 5.10), with DHS3 being the next best with a GRS of 8.25. The GRS statistics for the other models are significantly higher, ranging from 14.03 for BS6 to 18.23 for TFPM. The SFPM also has the lowest average absolute alpha (1.11% per month) and the lowest absolute alpha as a percentage of the average absolute premium (47.4%).

In line with Barillas and Shanken (2018) and other large-scale model comparisons (e.g., Ahmed et al., 2019; Swade et al., 2024), the BS6 model outperforms the other existing multifactor models, ranking second (after the SFPM) in producing lower average absolute alphas and squared alphas and third in its ability to explain the 17 anomalies. After the SFPM and BS6 models, the mispricing-based models (SY3 and SY4) and investment-based models (Q and Q5) perform best, with similar performance. While the DHS3 performs strongly in the GRS tests, the TFPM is the least effective in explaining the 17 anomalies, closely followed by the FF5, FFPM, and FF6 models.

TABLE 6 ABOUT HERE

9. Conclusion

This study investigates ten multifactor asset-pricing models in Canadian markets, offering insights into the factors relevant to explaining returns. While certain factors consistently show significance and yield positive alphas, others lack reliable average returns or are subsumed by alternative factors. The market factor consistently maintains relevance across models, producing significant alphas in the factor-redundancy regressions. In contrast, size factors such as SMB and SMB_P fail to generate reliable positive alphas and do not pass redundancy tests. In accordance with U.S.-based evidence, HML is redundant in explaining Canadian returns in the context of Fama and French's (2015) five-factor model, where CMA challenges it. The UMD factor is subsumed in models with R_{ROE} as a factor, suggesting a relationship between momentum and profitability.

In the Canadian context, not only do the mispricing-based models (SY3 and SY4) of Stambaugh and Yuan (2017) subsume the FF5 and FF6 models of Fama and French (2015, 2018), but the investment-based models (Q-factor and Q5) of Hou et al. (2015) and Hou et al. (2021) dominate the FF5, FF6, SY3, and SY4 models. Apart from these instances, no single model outperforms all other models in factor-spanning tests. The Q-factor and Q5 models are never subsumed because they include R_{ME}, R_{ROE}, and R_{EG}, which generate high abnormal premiums independently of the other 15 factors considered. Barillas and Shanken's (2018) composite six-factor model (BS6) is never subsumed because it comprises R_{ROE} and Asness and Frazzini's (2013) HML_M. These two well-timed factors usually shine in the battle of factors. Finally, the spanning tests always leave the DHS3 model of Daniel et al. (2020) unsubsumed because the post-earnings announcement drift factor (PEAD) not only delivers the highest average premium over 1991–2022, but this premium is never explained in any of the redundancy or spanning regressions. These findings underscore the importance of integrating multiple factors from diverse asset-pricing models to construct a more comprehensive and robust pricing framework.

Overall, our analysis of a wide-ranging set of old and new factors reveals the prominence of a six-factor asset-pricing model (SFPM) encompassing the market, size, monthly updated value, return on equity, expected growth, and post-earnings announcement drift factors for explaining asset return behavior in Canada. This study adds to the literature by providing insights into the relevance of and interactions among prominent factors. This emphasizes the need to assess the performance of asset-pricing models in diverse contexts and underscores the value of combining factors from different models to encapsulate stock market intricacies. This understanding is crucial for researchers seeking to construct comprehensive models and practitioners aiming to develop effective investment strategies in diverse financial landscapes. Future research can explore additional factors and refine existing models to further enhance our understanding of asset-pricing dynamics in various contexts.

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Table 1

Descriptive Statistics on the Factors

This table presents summary statistics on the 17 factors considered in this study. Panel A reports the monthly factor returns' mean and standard deviation, along with the robust Newey–West (1987) *t*-statistic testing of whether the average factor return is zero. The last three columns show the minimum, lower quartile, median, upper quartile, and maximum, while the second and third columns report the sample period and number N of observations for each factor. Panel B reports correlations (in percent) between the factors. Correlations higher than 40% in absolute terms are in boldface. The factors include the market factor ($R_M - R_F$), which is the value-weighted return of all Canadian stocks minus the monthly return on the 91-day Canadian Treasury bill. The market factor is shared by all models considered. We consider six factors associated with the TFPM, FFPM, FF5, and FF6 models (SMB, HML, UMD, RMW, and CMA). SMB is the size factor considering only the six size-value portfolios. HML is the value factor, UMD is the momentum factor, RMW is the probability factor, and CMA is the investment factor. We also present the results for the specific factors associated with the Q-factor and Q5 models (R_{ME}, $R_{I/A}$, R_{ROE}, and R_{EG}), the SY3 and SY4 models (SMB_P, MGMT, PERF, and UMO), the DHS3 model (FIN and PEAD), and the BS6 model (HML_M).

A. Factor Premiums

Variable	Period	N	Mean	*t*-value	SD	Minimum	Q1	Median	Q3	Maximum
$R_M - R_F$	1991:07 – 2022:12	378	**0.53**	**2.44**	4.05	-19.65	-1.49	0.86	3.18	11.61
SMB	1991:07 – 2022:12	378	0.36	1.51	4.32	-14.70	-2.34	0.15	2.67	36.96
HML	1991:07 – 2022:12	378	**0.88**	**2.70**	5.85	-19.89	-1.95	0.63	3.65	22.79
CMA	1991:07 – 2022:12	378	**0.61**	**3.31**	3.28	-13.03	-1.24	0.46	2.12	13.78
RMW	1991:07 – 2022:12	378	-0.06	-0.25	4.12	-20.31	-2.40	0.30	2.34	10.55
UMD	1991:07 – 2022:12	378	**1.08**	**3.85**	5.03	-26.37	-1.23	1.28	3.66	19.45
R_{ME}	1991:07 – 2022:12	378	**0.77**	**3.69**	3.79	-13.09	-1.48	0.76	2.84	23.65
$R_{I/A}$	1991:07 – 2022:12	378	**0.91**	**3.78**	4.46	-25.12	-1.71	0.86	3.26	17.11
R_{ROE}	1991:07 – 2022:12	378	**2.30**	**7.59**	5.64	-20.97	-0.30	2.42	5.62	19.69
R_{EG}	1991:07 – 2022:12	378	**2.06**	**4.37**	8.75	-39.61	-2.31	2.87	7.14	56.46
SMB_P	1991:07 – 2022:12	378	0.35	1.41	4.38	-15.73	-2.35	0.05	2.57	20.74
MGMT	1991:07 – 2022:12	378	**1.61**	**5.31**	5.68	-26.32	-1.82	1.30	4.56	25.24
PERF	1991:07 – 2022:12	378	**2.14**	**5.27**	7.34	-27.13	-1.28	2.53	6.06	42.62
UMO	1991:07 – 2022:12	378	**2.08**	**5.75**	6.57	-35.50	-1.31	2.48	5.75	25.41
FIN	1991:07 – 2022:12	378	**-0.11**	**-0.43**	4.71	-17.77	-3.05	-0.28	2.73	22.13
PEAD	1991:07 – 2022:12	378	**6.82**	**20.34**	6.92	-10.29	1.92	6.13	10.94	46.73
HML_M	1991:07 – 2022:12	378	**0.93**	**2.55**	6.64	-23.37	-2.49	0.50	3.87	34.77

Table 1 – *Continued*

B. Correlation coefficients

	TFPM, FFPM, FF5, and FF6					Q-factor and Q5				SY3 and SY4				DHS3		BS6
	SMB	HML	CMA	RMW	UMD	R_{ME}	$R_{I/A}$	R_{ROE}	R_{EG}	SMB_P	MGMT	PERF	UMO	FIN	PEAD	HML_M
$R_M - R_F$	0.08	-9.15	-24.67	-5.97	-25.69	-6.18	-15.66	-36.18	1.59	-3.20	-22.24	-29.07	-29.37	2.36	2.30	5.90
SMB		-7.69	3.75	-23.00	-6.57	**79.72**	-1.66	-23.10	-16.91	**74.69**	-2.43	-17.53	-14.93	0.83	-8.29	-7.02
HML			**51.82**	-36.97	6.07	15.54	**42.94**	18.15	-15.69	24.49	28.76	2.31	26.58	-7.69	-2.57	**57.20**
CMA				-26.07	17.66	7.40	**78.96**	19.20	-5.71	20.20	**69.81**	18.32	**56.96**	-9.61	20.28	26.96
RMW					12.69	-28.25	-7.71	15.92	26.04	**-46.62**	19.71	**40.78**	26.16	8.92	-5.88	-25.19
UMD						13.86	13.95	**57.49**	14.21	-6.49	29.39	**76.71**	**60.21**	-15.03	-1.39	**-50.52**
R_{ME}							1.98	2.79	-13.47	**77.05**	0.20	-1.17	-0.60	-6.61	-16.49	-7.08
$R_{I/A}$								13.83	5.28	13.88	**77.42**	21.37	**60.32**	-16.63	16.68	25.91
R_{ROE}									13.58	-23.44	26.54	**71.14**	**58.13**	-10.04	-0.90	-26.74
R_{EG}										-26.51	18.01	16.14	18.99	-9.80	5.94	-15.20
SMB_P											5.11	-20.82	-8.41	0.39	-5.19	22.69
MGMT												**40.74**	**78.45**	-17.18	17.94	9.52
PERF													**73.25**	-14.28	-1.27	**-42.92**
UMO														-15.62	8.60	-16.81
FIN															10.42	14.99
PEAD																11.73

Table 2

Factor Redundancy Tests

This table presents the results of the factor redundancy tests for ten asset-pricing models (TFPM, FFPM, FF5, FF6, Q, Q5, SY3, SY4, DHS3, and BS6). The redundancy tests involve determining whether a factor in each model generates a positive alpha (intercept) when regressed on the other factors of the same model. To serve as a reference, column 2 reproduces the average value of each factor [see Table 1]. Besides these average factor returns in the second column, we report only the alphas from the redundancy regressions. Below each estimate, we present (in parentheses) the robust t-statistic estimated using Newey and West's (1987) method. Details of the factor constructions are presented in Section 3. Panel A considers the market factor, Panel B the factors in TFPM, FFPM, FF5, and FF6 models, Panel C the factors in the Q-factor and Q5 models, Panel D the factors in the SY4 and SY3 models, Panel E the factors from the DHS3 model, and Panel F the distinct factor from the BS6 model. All alphas are reported in percent per month. Estimates that are significant at the 5% level are in boldface. The period covered is July 1991 to December 2022 (378 months).

Factor	Mean	Alpha relative to the other factors from the same asset-pricing model									
		TFPM	FFPM	FF5	FF6	Q	Q5	SY3	SY4	DHS3	BS6
					A. Market factor						
$R_M - R_F$	**0.53**	**0.59**	**0.81**	**0.74**	**0.89**	**1.23**	**1.18**	**0.93**	**1.00**	0.45	**1.29**
	(2.44)	**(2.46)**	**(3.52)**	**(3.27)**	**(4.05)**	**(6.18)**	**(6.07)**	**(4.27)**	**(4.78)**	(1.69)	**(5.92)**
					B. Factors from the TFPM, FFPM, FF5, and FF6 models						
SMB	0.36	0.41	0.49	0.44	0.17						**1.02**
	(1.51)	(1.71)	(1.74)	(1.70)	(1.56)						**(2.48)**
HML	**0.88**	**0.99**	**0.94**	0.44	0.43						
	(2.70)	**(2.56)**	**(2.48)**	(1.45)	(1.31)						
CMA	**0.61**			**0.45**	0.36						
	(3.31)			**(2.89)**	(1.91)						
RMW	-0.06			0.38	0.26						
	(-0.25)			(1.70)	(1.09)						
UMD	**1.08**		**1.24**		**1.08**						0.39
	(3.85)		**(4.69)**		**(4.05)**						(1.86)

Table 2 – *Continued*

Factor	Mean	Alpha relative to the other factors from the same asset-pricing model									
		TFPM	FFPM	FF5	FF6	Q	Q5	SY3	SY4	DHS3	BS6
C. Factors from the Q-factor and Q5 models											
R_{ME}	**0.77**					**0.18**	**0.87**				
	(3.69)					**(2.54)**	**(2.77)**				
$R_{I/A}$	**0.91**					**0.80**	**0.76**				0.26
	(3.78)					**(2.50)**	**(2.46)**				(0.91)
R_{ROE}	**2.30**					**2.45**	**2.26**				**1.96**
	(7.59)					**(8.41)**	**(7.01)**				**(7.97)**
R_{EG}	**2.06**						**1.58**				
	(4.37)						**(3.20)**				
D. Factors from the SY3 and SY4 models											
SMB_P	0.35							0.53	**0.58**		
	(1.41)							(1.77)	**(2.00)**		
MGMT	**1.61**								**0.95**		
	(5.31)								**(2.76)**		
PERF	**2.14**								**1.71**		
	(5.27)								**(5.13)**		
UMO	**2.08**							**2.39**			
	(5.75)							**(6.95)**			
E. Factors from the DHS3 model											
FIN	-0.11									-0.60	
	(-0.43)									(-1.66)	
PEAD	**6.82**									**6.82**	
	(20.34)									**(20.28)**	
F. Factors from the BS6 model											
HML_M	**0.93**										**1.47**
	(2.55)										**(4.37)**

Table 3

Factor-Spanning Tests

This table presents the results of the factor-spanning tests for 11 asset-pricing models (TFPM, FFPM, FF5, FF6, Q, Q5, SY3, SY4, DHS3, and BS6). The redundancy tests examine whether the factors from a given asset-pricing model generate positive alphas when regressed onto the factors of another asset-pricing model. To serve as a reference, column 2 reproduces the average value of each factor [see Table 1]. The figures in parentheses below each estimate are the robust Newey–West *t*-statistics. The factors are described in more detail in Section 3. Panel A tests the performance of the factors in the TFPM, FFPM, FF5, and FF6 models relative to the other asset-pricing models, Panel B focuses on the factors in the Q-factor and Q5 models, Panel C considers the factors in the SY3 and SY4 models, Panel D examines the performance of the factors in the DHS3 model, and Panel E considers the HML_M factor in the BS6 model. All alphas are reported in percent per month. Estimates that are significant at the 5% level are in boldface. The period covered is July 1991 to December 2022 (378 months).

Factor	Mean	Alpha relative to the factors of other pricing models									
		TFPM	FFPM	FF5	FF6	Q	Q5	SY3	SY4	DHS3	BS6
A. Performance of the factors from the TFPM, FFPM, FF5, and FF6 models											
SMB	0.36					0.16	0.17	0.22	0.15	**0.72**	
	(1.51)					(1.01)	(1.14)	(1.06)	(0.71)	**(1.98)**	
HML	**0.88**					-0.12	0.07	0.21	0.43	**1.03**	**-0.86**
	(2.70)					(-0.31)	(0.19)	(0.62)	(1.17)	**(2.31)**	**(-2.77)**
CMA	**0.61**	**0.43**	0.34			0.05	0.10	-0.02	0.07	-0.00	-0.08
	(3.31)	**(2.74)**	(1.80)			(0.35)	(0.78)	(-0.11)	(0.53)	(-0.01)	(-0.59)
RMW	-0.06	0.33	0.21			0.00	-0.16	-0.19	-0.41	0.26	0.20
	(-0.25)	(1.48)	(0.89)			(0.01)	(-0.63)	(-0.85)	(-1.89)	(0.82)	(0.69)
UMD	**1.08**	**1.25**		**1.08**		-0.20	-0.27	0.23	-0.04	**1.19**	
	(3.85)	**(4.69)**		**(4.05)**		(-0.73)	(-0.95)	(0.91)	(-0.22)	**(2.44)**	
B. Performance of the factors from the Q-factor and Q5 models											
R_{ME}	**0.77**	**0.42**	**0.25**	**0.49**	**0.32**			**0.49**	**0.42**	**1.39**	0.03
	(3.69)	**(4.05)**	**(2.33)**	**(4.77)**	**(3.27)**			**(3.10)**	**(2.92)**	**(3.66)**	(0.26)
RI/A	**0.91**	**0.69**	**0.59**	**0.16**	**0.18**			**-0.07**	**-0.01**	**0.15**	
	(3.78)	**(3.27)**	**(2.25)**	**(1.11)**	**(1.12)**			**(-0.32)**	**(-0.04)**	**(0.41)**	
RROE	**2.30**	**2.55**	**1.84**	**2.40**	**1.82**			**1.66**	**1.47**	**2.50**	
	(7.59)	**(9.36)**	**(7.23)**	**(9.85)**	**(7.25)**			**(6.83)**	**(6.75)**	**(4.54)**	
REG	**2.06**	**2.42**	**2.09**	**2.19**	**1.96**			**1.65**	**1.59**	**1.42**	1.93
	(4.37)	**(5.85)**	**(4.70)**	**(5.06)**	**(4.39)**			**(3.46)**	**(3.26)**	**(2.11)**	(3.69)

Table 3 – *Continued*

Factor	Mean	Alpha relative to the factors of other pricing models									
		TFPM	FFPM	FF5	FF6	Q	Q5	SY3	SY4	DHS3	BS6
C. Performance of the factors from the SY3 and SY4 models											
SMB_P	0.35	-0.12	-0.08	-0.04	-0.03	0.10	0.21			0.60	-0.17
	(1.41)	(-0.83)	(-0.54)	(-0.32)	(-0.24)	(0.64)	(1.27)			(1.89)	(-1.04)
MGMT	**1.61**	**1.53**	**1.18**	**0.75**	**0.61**	0.48	0.35			0.61	0.46
	(5.31)	**(5.29)**	**(3.26)**	**(4.45)**	**(3.55)**	(1.77)	(1.32)			(1.17)	(1.84)
PERF	**2.14**	**2.55**	**1.21**	**2.06**	**0.98**	-0.01	-0.09			**2.33**	0.28
	(5.27)	**(6.60)**	**(5.14)**	**(6.44)**	**(4.22)**	(-0.03)	(-0.32)			**(3.61)**	(1.31)
UMO	**2.08**	**2.16**	**1.27**	**1.41**	**0.78**	0.12	0.00			**1.60**	0.28
	(5.75)	**(6.76)**	**(4.89)**	**(5.45)**	**(3.40)**	(0.44)	(0.02)			**(3.39)**	(1.13)
D. Performance of the factors from the DHS3 model											
FIN	-0.11	-0.06	0.11	-0.05	0.11	0.30	0.37	0.16	0.24		0.03
	(-0.43)	(-0.24)	(0.42)	(-0.17)	(0.38)	(0.99)	(1.23)	(0.50)	(0.80)		(0.10)
PEAD	**6.82**	**6.88**	**6.91**	**6.63**	**6.70**	**6.82**	**6.78**	**6.59**	**6.60**		**6.59**
	(20.34)	**(18.57)**	**(14.16)**	**(21.01)**	**(17.16)**	**(11.92)**	**(12.55)**	**(16.41)**	**(16.50)**		**(12.38)**
E. Performance of the factors BS6 model											
HML_M	**0.93**	0.27	**1.18**	0.30	**1.11**	**1.47**	**1.64**	**1.10**	**1.38**	0.24	
	(2.55)	(0.84)	**(4.06)**	(1.07)	**(4.62)**	**(3.51)**	**(3.88)**	**(2.73)**	**(3.69)**	(0.42)	

Table 4

GRS Statistics from the Factor-Spanning Tests

This table presents the results of the joint tests of whether the factors unique to one model produce nonzero alphas with respect to another model. For each model, the unique factors considered as regressands are in parentheses in the first column, while the common factors excluded for all models are the market and size factors. We report the Gibbons–Ross–Shanken (1989) *F*-test of whether a given model produces zero alphas for the factors of a benchmark model, with the associated robust *p*-value using the Newey–West (1987) approach in brackets. The multifactor models considered are the six-factor model (FF6) of Fama and French (2018), the Q5 model of Hou, Mo, Xue, and Zhang (2021), the mispricing-based four-factor model (SY4) of Stambaugh and Yuan (2017), the three-factor behavioral model (DHS3) of Daniel, Hirshleifer, and Sun (2020), and the six-factor model (BS6) of Barillas and Shanken (2018). The factors are described in more detail in Section 3. Estimates that are significant at the 5% level are in boldface. The sample period is from July 1991 to December 2022 (378 months).

Models being tested	Benchmark models				
	FF6	Q5	SY4	DHS3	BS6
FF6 (HML, RMW, CMA, UMD)		0.77 [0.54]	1.91 [0.11]	**10.73** **[0.00]**	**5.89** **[0.00]**
Q5 ($R_{I/A}$, R_{ROE}, R_{EG})	**28.36** **[0.00]**		**22.12** **[0.01]**	**28.87** **[0.00]**	**19.05** **[0.00]**
SY4 (MGMT, PERF)	**16.20** **[0.00]**	2.39 [0.09]		**20.59** **[0.00]**	**3.55** **[0.03]**
DHS3 (FIN, PEAD)	**188.66** **[0.00]**	**185.81** **[0.00]**	**175.57** **[0.00]**		**173.88** **[0.00]**
BS6 (UMD, $R_{I/A}$, R_{ROE}, HML_M)	**38.76** **[0.00]**	**14.54** **[0.00]**	**18.01** **[0.00]**	**24.29** **[0.00]**	

Table 5

Redundancy of the Factors in the Proposed Six-Factor Pricing Model (SFPM)

This table presents the results of the factor redundancy tests for the SFPM. The redundancy tests involve determining whether a factor in the SFPM generates a positive alpha (intercept) when regressed on the other five factors of the SFPM. We report the intercept and five slope coefficients with the associated robust t-statistic estimated using Newey and West's (1987) method in parentheses below for each of the six redundancy regressions. Details of the factor constructions are presented in Section 3. Estimates that are significant at the 5% level are in boldface. The period covered is July 1991 to December 2022 (378 months).

Factor	Alpha	Market	R_{ME}	HML_M	R_{ROE}	R_{EG}	PEAD	R^2 (%)
Market	**1.108**		-0.047	-0.023	**-0.271**	0.025	0.008	13.85
	(4.51)		(-0.89)	(-0.51)	**(-6.47)**	(0.91)	(0.28)	
R_{ME}	**1.495**	-0.046		-0.039	0.006	-0.059	**-0.081**	5.09
	(3.60)	(-0.92)		(-0.66)	(0.10)	(-1.27)	**(-2.10)**	
HML_M	**1.230**	-0.066	-0.114		**-0.307**	-0.100	0.108	10.53
	(2.77)	(-0.51)	(-0.71)		**(-2.75)**	(-0.99)	(1.41)	
R_{ROE}	**2.472**	**-0.488**	0.011	**-0.198**		0.068	0.017	20.31
	(4.84)	**(-4.87)**	(0.10)	**(-3.08)**		(1.49)	(0.23)	
R_{EG}	**1.509**	0.129	-0.313	-0.182	0.193		0.067	6.02
	(2.21)	(0.84)	(-1.42)	(-1.03)	(1.34)		(0.68)	
PEAD	**6.739**	0.026	-0.272	0.126	0.032	0.043		4.21
	(13.14)	(0.28)	(-1.95)	(1.34)	(0.23)	(0.66)		

Table 6

Summary Statistics for the Absolute Performance of Various Pricing Models

This table examines the ability of various multifactor pricing models to explain a set of 17 return anomalies highlighted in the literature. For each of the 17 characteristics related to these anomalies, we compute the value-weighted returns of the bottom and top deciles and take the differential as the anomalous return to be explained. The multifactor models considered are the TFPM of Fama and French (1993), the FFPM of Carhart (1994), the FF5 and FF6 models of Fama and French (2015), the Q and Q5 models of Hou, Mo, Xue, and Zhang (2021), the SY3 and SY4 models of Stambaugh and Yuan (2017), the DHS3 model of Daniel, Hirshleifer, and Sun (2020), the BS6 model of Barillas and Shanken (2018), and the SFPM proposed in this study. For each set of 17 regressions, the table presents the number of significant intercept coefficients (alphas) at a 5% level (#SIG), the average absolute monthly alphas ($A|\alpha_i|$) in percent, the average absolute *t*-values ($A|t|$), the average absolute alphas over the average absolute value of $\bar{r}_i$ (the average return on the spread portfolio i), the average squared alpha over the average squared $\bar{r}_i$, and the average regression R^2 adjusted for degrees of freedom ($A(R^2)$). In the last two columns, we report the F-statistic and associated *p*-value for the GRS test, which assesses whether all 17 estimated alphas are jointly zero. The period covered is July 1991 to December 2022 (378 months).

Model	#SIG	$A\|\alpha_i\|$	$A\|t\|$	$\frac{A\|\alpha_i\|}{A\|\bar{r}_i\|}$	$\frac{A(\alpha_i^2)}{A(\bar{r}_i^2)}$	$A(R^2)$	GRS	*p*-value
TFPM	12	2.192	3.060	0.939	0.950	9.14%	18.234	0.000
FFPM	9	1.813	2.565	0.776	0.754	13.61%	17.515	0.000
FF5	8	1.915	2.661	0.820	0.839	15.55%	17.466	0.000
FF6	8	1.621	2.304	0.694	0.688	19.87%	17.023	0.000
Q	7	1.567	2.087	0.671	0.673	17.95%	15.505	0.000
Q5	7	1.545	2.027	0.662	0.668	19.85%	15.917	0.000
SY3	3	1.447	2.037	0.620	0.621	12.23%	16.436	0.000
SY4	4	1.555	2.204	0.666	0.632	18.14%	16.540	0.000
DHS3	8	1.853	2.001	0.793	0.603	6.77%	8.254	0.000
BS6	3	1.249	1.688	0.535	0.521	22.92%	14.029	0.000
SFPM	2	1.108	1.181	0.474	0.166	22.60%	5.095	0.000